The Quantum Theory

Concerning the early years of this fundamental concept of modern physics: how Max Planck formulated it at the turn of the century and how others enlarged it up to 1923

by Karl K. Darrow

THIS is the history of a physical theory which began in 1900 by taking over a small province of physics and now has extended its empire over almost the whole of the sciences of physics and chemistry. More precisely, it is half of the history, for it carries the story only to 1923, which happens to be about as far along the road as the nonphysicist can travel easily. The story is that of the quantum theory, and this account will relate the main events in its early history.

Usually a doctrine or discovery in science does not spring unheralded and full-blown from a single man's brain; behind the reported founder of the doctrine there are likely to be others who partly anticipated him, and the origin of the idea may be lost in the mists. This cannot be said of quantum theory. The recognized founder was the actual founder, and there was no one behind him. His name was Max Planck; he was born in Germany in 1858, and he died there in his 89th year in 1947. Tacitus has said of some Roman character that he was *felix opportunitate mortis*—he was lucky to die when he did. Not so for Planck: he suffered grievously in the two world wars, losing a son in each and his house and his library in the second; he would have had a happier life if he had died in 1914. He would also have been equally famous, for his grand idea had come at the turn of the century.

Planck achieved his doctorate by a thesis on an experiment in the diffusion of hydrogen through palladium. This was the only experiment he ever performed: one may suspect that it was imposed on him rather than chosen by him. Even before he won his doctorate, he had devoted himself to the theoretical study of the foundations of thermodynamics. He saw rather more deeply into the second law of thermodynamics than anyone before him, and his textbook on thermodynamics is still a classic.

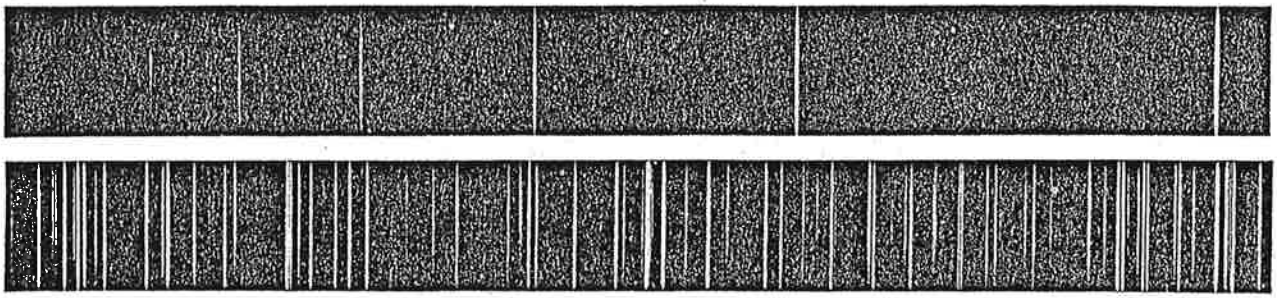
It was by way of thermodynamics that Planck came to the quantum theory. Thermodynamics is a hard subject, and Planck arrived at the quantum theory by the hard way. When he entered upon thermodynamics, it was a science of heat in matter. Planck extended it to light.

The Cavity Problem

Imagine a cavity inside a solid body. There is a small perforation in the wall of the cavity, out of which light can come. It must be large enough so that we can get enough light out of it to analyze with a spectroscope; it must also be small enough so that the light which comes out will be a fair sample of what the light inside would be if the perforation were not there. We have therefore to assume that one and the same perforation can be neither too large nor too small. This sort of double-barreled assumption is frequently made in physics, and in this case at least it has not yet led us into any trouble.

First let us suppose that there is some gas inside the hollow. For definiteness let the gas be helium, which is a nice monatomic gas and nearly a perfect gas. The gas has energy, which is the kinetic energy of its atoms. This kinetic energy we know to be proportional to the absolute temperature. We also know that the energy is proportional to the number of atoms. If I pump half of the gas out of the cavity while the temperature remains the same, the energy of the gas in the hollow is reduced to one-half; if I pump extra helium into the cavity so as to double the quantity, I double the energy.

Let us now imagine that all of the gas is pumped out. The cavity is still not empty, for it still contains light, in fact, just the same amount of light as while the gas was there. Since the cavity contains light, it contains the energy of that light. This energy is proportional, however, not simply to the temperature but to the *fourth power* of the temperature. There is also a much more important difference between light and helium: namely, whereas the energy of the helium in the cavity is proportional both to the absolute temperature and to the amount of helium, the energy of the light depends only on the temperature. There is no point in saying that the



SPECTRA OF HYDROGEN show bright lines which correspond to energy transitions in the hydrogen atom and molecule. The brighter lines in the top spectrum are emitted by atomic hydrogen; the more complex lines in the bottom spectrum, by the hydrogen molecule. The spectra were made at The Johns Hopkins University.

energy of the light is proportional to the amount of light, for the amount cannot be varied. There is no pump by which I can pump out half of the light and leave the rest. I could indeed increase the amount of light in the hollow by shining a searchlight through the perforation, but this would make no lasting difference. The beam of the searchlight would be absorbed in the walls of the cavity and heat them up; as soon as I turned off the searchlight, the temperature would start to revert to its original value, and when it got back to its initial value there would be the same amount of light in the cavity as before.

The light emerging through the perforation has a perfectly distinctive spectrum, and this we have already assumed to be identical with the spectrum of the light inside the cavity. On a graph in which energy is plotted against frequency, the spectrum has a single broad hump, meaning that the energy is greatest at the middle wavelengths in the spectrum and least at the highest and lowest frequencies. This spectrum has several names. Planck called it the "normal spectrum." Germans generally call it the spectrum of "cavity radiation," which is the best name. English-speaking people usually call it the "black-body spectrum," because a perfectly black body would emit light of this composition. (It seems paradoxical that a black body should emit light, but it is no paradox at all when we remember the distinction between emission and reflection. The practical definition of blackness is that it is the quality of not reflecting light. A black body does not reflect, but it may emit; in fact, it is the most powerful possible emitter of light. We ordinarily look at bodies—black, white or colored—when they are reflecting light but are not hot enough to emit an appreciable amount of visible light. At temperatures of incandescence a black body would look brightest of all.)

Little Resonators

Planck set out to explain the black-body spectrum. He postulated that all solid bodies, and therefore the walls of the cavity, contained little oscillators—he called them resonators—of every imaginable frequency. By resonators Planck meant small electrified particles which wiggled to and fro. (A few years later he identified them as vibrating electrons, but in 1900 the electron had only just been discovered and was far from well established in physics.) It followed that in any range of frequencies, however narrow, there must be resonators having frequencies in that range. Take, for instance, the bright yellow spectrum-line of sodium which is designated by the letter D. We are to imagine that there are resonators having frequencies within this narrow range. All of them are emitting yellow light into the cavity, and yellow light is coming right back out of the cavity and bathing them and stimulating them to continued oscillation. There is an equilibrium between the energy of the resonators and the energy of the yellow light. So it is with the energy of blue light and the resonators which have the frequency of blue light, and so it is throughout the entire spectrum. If by any valid line of reasoning one could arrive at the energy which the resonators of any frequency have when they are in equilibrium with the light, one would be able to arrive by one simple further step at the energy which the light of any frequency has when it is in equilibrium with the resonators. This would be the answer to Planck's question, for it would give the black-body spectrum.

There was an answer to this question before Planck, but the answer was a disaster. This "classical" answer was that all of the resonators have the same energy, no matter what their frequency. From this it followed that the quantity of light in the cavity would be infinite, and since the quantity of light in the cavity actually is finite, the theory was sunk.

The flaw in the classical theory lay in one of its assumptions—indeed, a very plausible one—to wit: that any resonator may have any amount of energy; in other words, that it may go continuously up or down the energy-scale through all levels, like a man walking up or down a ramp. Planck replaced the ramp by a ladder, with its rungs, as in any practical ladder, at equal intervals. His assumption was that the resonator must always be on one or another of the rungs, and if it proceeds up or down the ladder, it must proceed by jumping from rung to rung. That is to say, it may absorb or emit energy only in certain discrete units, represented by the equal spacing from rung to rung of the ladder. Planck then affirmed that the spacing between the rungs of the ladder was not the same for all resonators but must be proportional to the frequency of the resonator in each case. He designated the spacing, or energy unit, as $h\nu$, and said that h is a universal constant, ν being the frequency of the resonator.

Planck's substitution of the ladder for the ramp not only avoided the disaster of infinity but yielded the actual form of the black-body spectrum. To explain just why it did so, it would be necessary to define entropy; to quote what is called Boltzmann's relation between entropy and probability; to define probability in a very special and peculiar way; to derive the formulas for the probability and the entropy of a flock of resonators, among which parcels of energy of the amount $h\nu$ are distributed in the most probable way; to say that the derivative of the entropy with respect to the energy is the reciprocal of the absolute temperature; to show that when all this is done, one arrives at a formula which gives the energy of the resonators of any chosen frequency ν as a function of the temperature, and to conclude by showing that out of this formula comes the black-body spectrum which is ratified by experiment. This skeleton of the argument shows that the argument itself can find no place in an article designed for people who are not mathematical physicists. It will certainly be conceded that Planck got to the quantum theory the hard way.

The constant h is known as Planck's constant. Its value must be determined by experiment; it is not prescribed by theory. It has been measured in a variety of ways, and all of the measurements agree. Planck may be characterized as the man who put h into physics. People who drop the h are sometimes called cockneys; in this sense the physics of the period before Planck may be called cockney physics. If the physicists of that period could return for a glimpse of our physics, they might say that it is cockeyed physics—and yet ours is right, and they would be wrong.

I have said that the rungs of the ladder—the "levels," as they are more generally called—are spaced at intervals $h\nu$; I have not said where the ladder starts. In Planck's original theory the bottom rung was at zero; the energy-values permitted to a resonator were 0, $h\nu$, $2h\nu$, $3h\nu$, and so on. Later Planck revised his theory and located the bottom rung at the energy-value $(\frac{1}{2})h\nu$, so that the subsequent rungs followed along at $(\frac{3}{2})h\nu$, $(\frac{5}{2})h\nu$, and so on. So long as we concern ourselves with black-body radiation alone, it makes no difference where we start, but other phenomena confirm Planck's second choice.

Particles of Light

If light is emitted and absorbed only in units of size $h\nu$, the obvious next step is to infer that light travels around in units or parcels or particles of energy $h\nu$. But this is a step that Planck himself did not take. According to some of those who knew him and to the evidence of his own writings, Planck was a revolutionary of a very conservative kind. He took one radical step when it was necessary, but he was not daring enough to try to revive the corpuscular theory of light, and this is understandable in view of its state at the time.

In 1900 the corpuscular theory of light was very dead. Newton's idea that light consisted of particles had yielded in the 19th century to the wave theory, and almost anyone who might have attempted to restore the corpuscular theory would have run grave danger of being rated a crank. To resurrect it there was needed a man possessed of (a) prestige, (b) courage, (c) knowledge

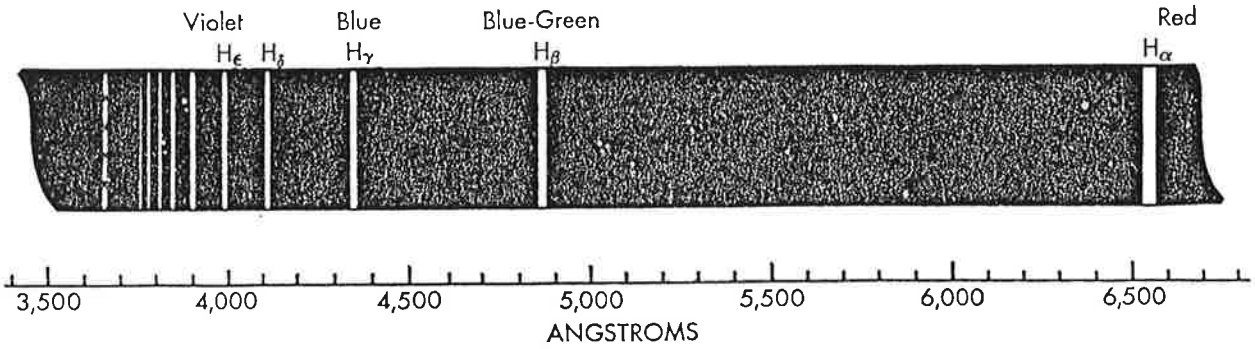


of Planck's theory and (d) knowledge of certain experiments which had not yet been performed when Planck put forth his theory. There was such a man, and his name was Einstein.

The new experiments were those on the photoelectric effect. This effect consists in the emission of electrons from metals under the influence of light. Think of a beam of light playing on a piece of metal—sodium, for example. The beam of light, Einstein imagined, consists of corpuscles of energy $h\nu$. Some of them are reflected from the surface of the metal; these do not concern us. Others enter the metal, and one of these may be absorbed in such a way that it vanishes completely, yielding up its entire energy $h\nu$ to a single electron in the metal. The electron thereby gains enough speed and energy to spring out of the metal, and it becomes a "photoelectron." Once it is out, we can measure its kinetic energy.

This energy of course is something less than $h\nu$. The electron has had to spend a certain amount of its energy in order to leave the metal; we might describe this as the fee it pays at the surface of the metal for its exit-visa. We denote this fee by the letter W; thus the energy the electron has left when it gets out to where we can observe it is $h\nu$ minus W. If we could measure only photoelectrons released by one frequency of light we should be unable to separate W from $h\nu$. But we can experiment with beams of light of many different frequencies, and can find the kinetic energy of electrons emitted under the impact of each frequency. The theory says that if we plot the electron-energy E against the frequency ν, the points should lie along a straight line, described by the equation: $E = h\nu - W$. Experiments confirm that the points *do* lie along a straight line, and that the slope of this line is h, Planck's constant. As for W, it can be determined too; but we will leave that constant to the people who are most interested in it—the students of solid-state physics.

The discovery of this equation and the revival of the corpuscular theory of light are milestones in the history of physics. Einstein's theory inspired many famous experiments; one of the chief experimenters was Robert A. Millikan. It must be remarked that even Einstein was not bold enough to come out for the corpuscular theory of light unreservedly. He qualified his idea with a singular word, which I think proper to call a hedging word—"heuristic." According to the *New International Dictionary* heuristic means "serving to discover or reveal—applied to methods of demonstration which are persuasive rather than compelling, or which lead a person to find out for himself." I interpret that Einstein meant to convey that light acts in the photoelectric effect as though it



SPECTRAL LINES of atomic hydrogen are mapped against their color (*top*) and wavelength in Angstroms (*bottom*). This is the celebrated Balmer series, one of several such spectral series which reflect the energy transitions of the hydrogen atom. The study of these series led to Bohr's model of the atom (*see next page*).

were corpuscular, without committing himself further.

Today there is no longer any reason to hedge. The corpuscular theory of light has been re-established. In its reincarnation it is the first child of the quantum theory. The art by which it is combined with the wave theory is also a triumph of quantum theory, but this is a part of the later history. A remark about language is appropriate here. It must come as quite a surprise to a student of language to learn that "quantum theory" and "quantum mechanics" and "quantum number" are terms very common in physics, while the word "quantum" itself has practically vanished. Quantum, first introduced to denote the quantity $h\nu$, later came to be applied to the corpuscles of light. In this latter sense it has been superseded by a much better word—photon.

Vibrating Atoms

The next province of physics to be invaded by the quantum theory was the province of heat in matter. The cavity exhibits heat in the form of radiant energy. Heat in solids is identified with the vibrations of the atoms. Since Planck had scored quite a resounding success with the assumption that oscillators interchange energy with radiation in discrete units, nothing could be more natural than to try out the same idea on the oscillating atoms in solids. Like other ideas which seem very natural after somebody else has thought of them, this idea took several years to emerge. Again we must salute Einstein as the theorist, and on this occasion Walther Nernst of Germany was the leading experimenter.

The classical theory (note that in physics the term "classical" is often applied to the last-but-one theory in the field) said that the heat content of a solid is proportional to its absolute temperature. Another way of stating the theory is that the specific heat of a solid, which is the amount of heat needed to impart a unit increase in temperature to a unit mass of the substance, should be independent of the temperature at which the solid happens to be when the heat is applied. Actually the amount of heat needed to raise the temperature of a solid by a unit (say one degree) falls off as its temperature declines. As experimentalists made progress toward lower and lower temperatures, the departures from the classical theory became more and more striking. The quantum theory arrived in time to take care of them. It was found that the ladder of levels is the same for the vibrating atom as for Planck's electrical resonator. Since there is no rung below $(\frac{1}{2})h\nu$, the quantum theory tells us that a vibrating atom can never have a lesser energy than this—not even at absolute zero temperature, where according to classical theory the atoms should be standing absolutely still. This singular result of quantum theory has been attested by experiment.

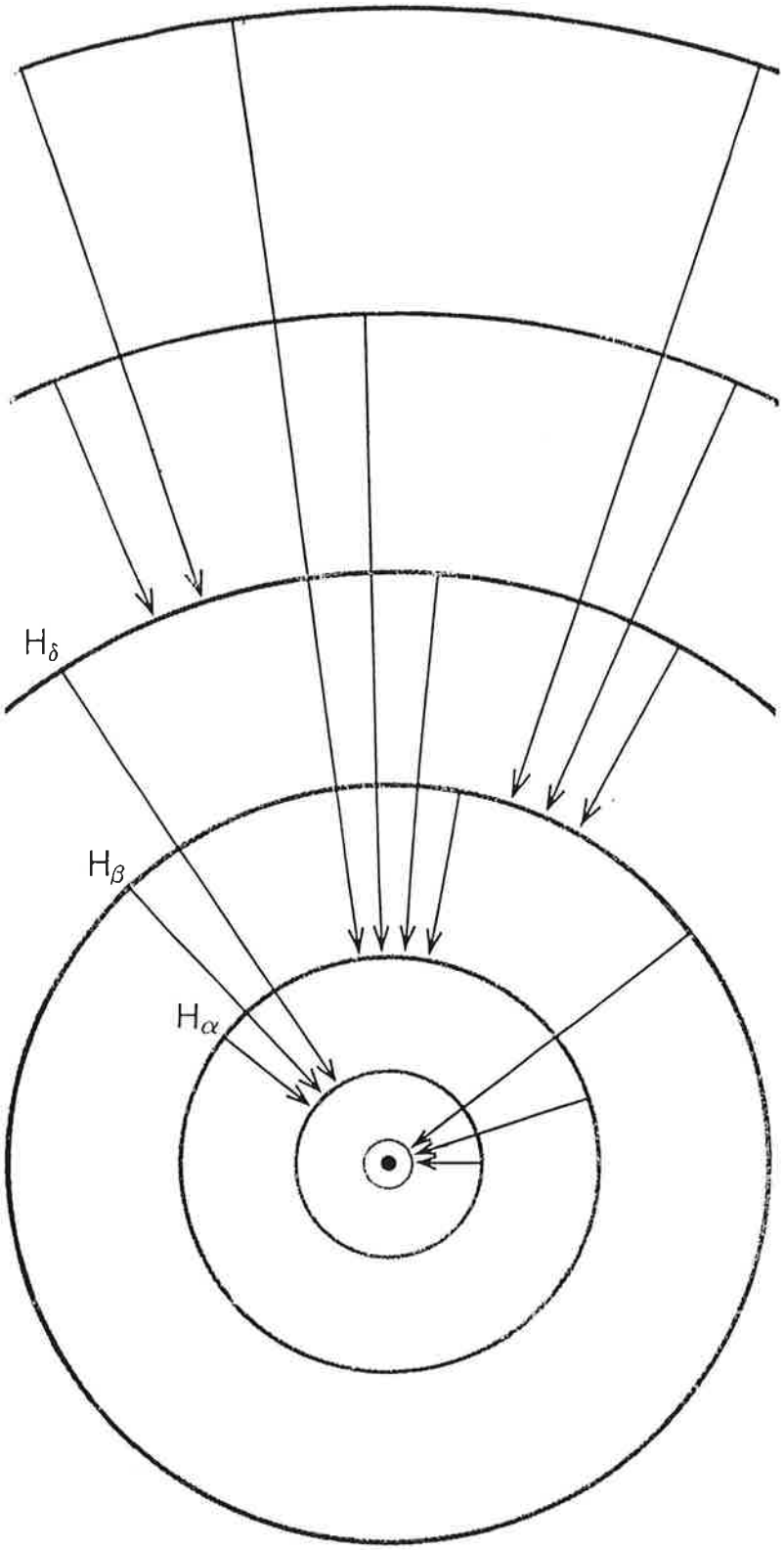
Another type of substance which defers to quantum theory is the gas whose molecules contain two atoms, such as hydrogen, H_2. The energy picture of a monatomic gas such as helium is simple: the kinetic energy of its atoms is purely the energy of traveling motion (*i.e.*, without the complication of rotation of the atoms themselves), and the quantum theory has nothing new to say about it—not at least until the temperature falls extremely low. But the molecules of a diatomic gas not only move about as traveling bodies but also rotate around the axis connecting the two atoms—think, if you will, of a dumbbell spinning end over end. The heat content of such a gas is partly the kinetic energy of traveling motion and partly the kinetic energy of rotatory motion. It was no slight step forward to imagine that rotatory energy may be limited to definite energy-values just as vibratory energy is. Starting from this advance, the quantum theory, which had already extended its domain over the fields of black-body radiation, the photoelectric effect and specific heat, was now to conquer the atom.

Before we begin that part of the story, I should make clear what is meant by the statement that rotatory energy is limited to certain definite discrete values, or, as the physicist says, is "quantized." Rotatory motion involves the concept of angular momentum, one of the most important in physics. For a particle revolving in a circular orbit, the angular momentum is the mass times the angular velocity; for a rigid body rotating around an axis passing through itself, the angular momentum must be found by calculation; often in quantum physics it appears as a quantity which is easy to measure but impossible to separate into mass and angular velocity.

The law of quantization for angular momentum is the simplest of all laws. In this ladder the unit of spacing between the evenly spaced rungs is $h/2\pi$, and the scale of permitted values of angular momentum usually runs: 0, $h/2\pi$, $2h/2\pi$, and so on. It may be surprising to find Planck's constant h reappearing in this new role. First it was a factor by which the frequency of a vibrator must be multiplied in order to get the spacing between the permitted energy-values; now, multiplied by $1/2\pi$, it is the interval between the permitted values of angular momentum of a rotating body. It can play both these roles because it has what physicists call the correct dimensions for both. Planck's constant has the dimensions of energy divided by frequency, and these are also the dimensions of angular momentum.

Bohr

The quantum theory's conquest of the atom began in the year 1913 (remembered with nostalgia by those who are old enough as the last year of the good old times), and the leader in this conquest was a young Dane by the name of Niels Bohr, who came to the University of Manchester to study with the celebrated Ernest Rutherford. Rutherford had just established the conception that the atom is a sort of miniature of the solar system, in which the planets are represented by negative electrons and the central sun by a positively-



BOHR MODEL postulated a hydrogen atom in which an electron traveled around a nucleus in any one of several "permitted" orbits. When the electron fell from an orbit of higher energy to one of lower energy, it emitted light of a characteristic wavelength. In this diagram the electron orbits are indicated as circles or sections of them. Each group of arrows represents a group of energy transitions which give rise to a spectral series. The three labeled arrows at the left refer to the Balmer series shown on the preceding page. The labels correspond to three spectral lines in that drawing.

charged and massive nucleus, around which the electrons revolve in orbits as the planets revolve around the sun. In the simplest case, the hydrogen atom, there is only one electron. Rutherford did not prescribe the particular orbit in which this electron should revolve. To prescribe it required a revolutionary new idea, and Bohr was the man who provided this idea.

The orbit of the electron around the nucleus, like that of a planet around the sun, should be an ellipse, for in both cases the force is an inverse-square force. A circle is a particular case of an ellipse; we will consider only circular orbits. A hydrogen atom, with its electron revolving in a circular orbit about its nucleus, can be regarded as a wheel. It is a peculiar kind of wheel, since it has no spokes and the rim is vacant except for the small region occupied by the electron, but it possesses the major property of a wheel: angular momentum. Bohr made the assumption that the angular momentum of a hydrogen atom is quantized, and that the electron is permitted to revolve in certain orbits which correspond to the integer multiples of $h/2\pi$. In its "normal state" the hydrogen atom has its electron revolving in the circle for which the angular momentum is $h/2\pi$. Its other permitted orbits are those for which the values of the angular momentum are $2h/2\pi$, $3h/2\pi$, and so on. Each of these other orbits, Bohr postulated, represents an "excited state" of the hydrogen atom.

The energy of the hydrogen atom in its normal state is $-R$; R being a constant of which Bohr computed the theoretical value. The general formula which expresses the various energy-values permitted to the hydrogen atom is $E_n = -R/n^2$. In its first excited state, represented by $n=2$, its energy is $-R/4$; in the second excited state it is $-R/9$, and so on. One may be puzzled by the fact that all of these energy-values are negative. This is because we are reckoning energy from a zero which corresponds to the state in which the hydrogen atom is completely torn apart, with the nucleus and the electron infinitely far from each other. It would seem more rational to reckon energy from a zero which corresponds to the normal state of the hydrogen atom, but this actually makes the formula more complicated.

Bohr went on to propose a second revolutionary idea, without which the first would have been of little use. Imagine a hydrogen atom in, let us say, the second excited state—the one for which the energy is $-R/9$. Suppose that the electron transfers itself into the orbit corresponding to the first excited state—the one for which the energy is $-R/4$. The atom now loses the difference in energy $(R/4 - R/9)$. What happens to this energy? According to Bohr's second idea, it leaves the atom, in the form of a

single photon. Accordingly, there should be, in the spectrum of hydrogen, a line of which the frequency is equal to $(R/4 - R/9)/h$. Moreover, there should be lots of other lines, and all of their frequencies should be calculable by inserting various integer values for m and for n in the general formula:

$$\nu = (R/m^2 - R/n^2)/h$$

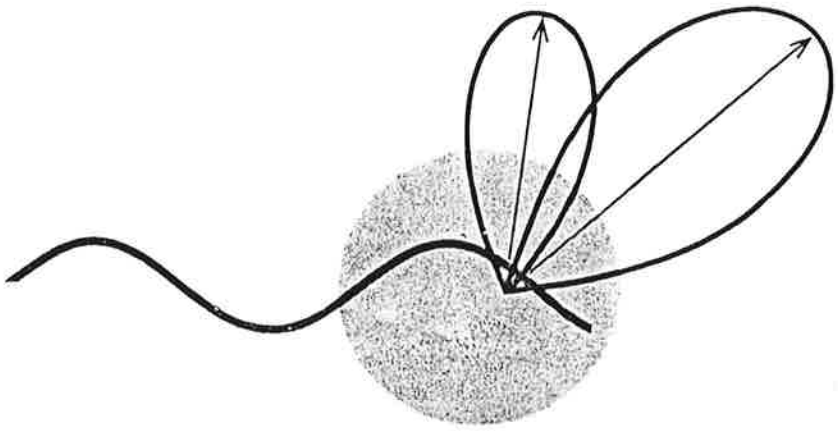
Well, this is a correct description of the actual spectrum of hydrogen, and the theoretical value assigned by Bohr to the constant R agrees with the value derived from experiments. There is nothing in the world which impresses a physicist more than a numerical agreement between experiment and theory; and I do not think that there can ever have been a numerical agreement more impressive than this one, as I can testify who remember its advent.

This is a good place to interpolate that one of the most curious features of quantum theory is that one and the same formula can be derived from markedly different postulates. The picture of the hydrogen atom today differs somewhat from the one Bohr proposed, and yet the new picture leads to the same formula for the spectrum of hydrogen and to the same value of R. The conjurers of quantum theory get the same rabbit out of more than one hat.

Spins

Now let us consider some other wheels. The first of these is the electron. The electron, besides revolving around the nucleus, possesses an angular momentum of its own, and we liken it therefore to a wheel. It may be visualized as a rigid body spinning upon its axis, but this is a rather dangerous analogy, for it leads one to inquire what the electron's angular velocity is, and no one has ever been able to answer this question—indeed, it is very likely unanswerable. The electron's angular momentum is quantized, and in the simplest conceivable fashion: this is a ladder with only one rung. The angular momentum of the electron is fixed forever at the single value $(\frac{1}{2})h/2\pi$.

The electron is considered to be an elementary particle; I call it a "structural elementary particle," meaning a particle which is used in our models of atoms and of the nuclei of atoms. There are two other structural elementary particles: the proton and the neutron. The proton is the nucleus of the commonest and lightest kind of hydrogen atom. The neutron cannot serve as the nucleus of an atom when it is by itself, but it combines with protons and with other neutrons to form composite nuclei. These three structural elementary particles have one quality in common, and only one. Each has the same one-rung ladder of angular momentum; each has the unalterable angular momentum $(\frac{1}{2})h/2\pi$.

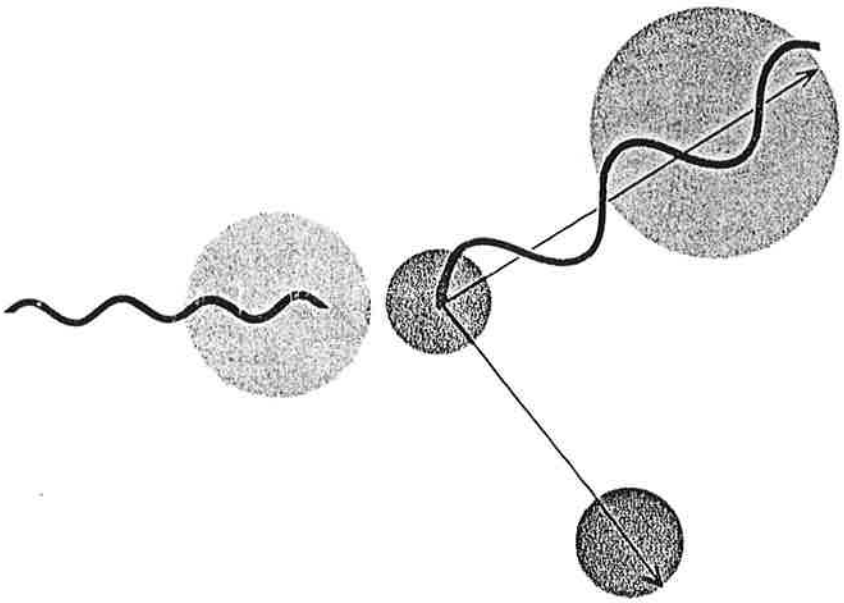


PHOTOELECTRIC EFFECT occurs when a photon (*wavy line*) encounters an atom (*gray circle*) and knocks out one of its electrons (*arrows*). A low-energy photon causes the electron to leave the atom at a steep angle; a high-energy photon, at a shallower angle. The curves around the arrows show the angular distribution of electrons resulting from many such photons.

The most important feature of angular momenta, on the atomic and the subatomic scale, is the so-called law of composition, meaning the composite angular momentum of a system consisting of two or more particles. I will illustrate this law by three examples.

The first is the common light hydrogen atom, containing one proton and one electron. The spins of these two particles are always either parallel or antiparallel to each other. Thus there are two, and just two, kinds of light hydrogen atoms. The difference in their properties is very slight indeed, and yet it can be detected by a spectroscope operating in the microwave region.

My second example is afforded by the hydrogen molecule, consisting of two common hydrogen atoms. This has two protons and two electrons. The two electrons always have their spins pointed antiparallel to each other. The two protons may be pointed in the same or opposite directions to each other; there are no intermediate cases. Thus there are two, and just two, kinds of molecular hydrogen. Their physical properties differ appreciably, and they can be separated from each other. They even have



COMPTON EFFECT occurs when a high-energy photon (*wavy line and gray circle at left*) encounters an electron (*smaller circle in center*). The electron is knocked away at an angle (*lower right*). The wavelength of the photon is decreased by the amount of energy imparted to the electron (*upper right*).

different names: ortho-hydrogen and para-hydrogen.

My last example is provided by the simplest of all composite nuclei—the deuteron. This is the nucleus of the isotope which used to be called heavy hydrogen, though this name is waning now that a still heavier isotope is known. The deuteron consists of one proton and one neutron. In every deuteron the spins of the proton and the neutron are parallel. This is a sharper restriction than prevails in my other two examples. The theorists believe that the spins of the proton and the neutron have no objection *per se* to setting themselves antiparallel, but that the forces between these two particles are such that in the antiparallel orientation the proton and the neutron just cannot stick together.

Notice now what would be the consequence if this law of composition of angular momenta did not exist. There would be not just two kinds of hydrogen atom but an infinite variety of kinds, though it is true that they would not differ appreciably in any significant quality. There would be not just two kinds of molecular hydrogen but an infinity of different kinds, with properties lying all the way between those of para-hydrogen and those of ortho-hydrogen; these would spoil the distinctiveness of molecular hydrogen. There would also be a wide variety of kinds of deuteron.

Follow this idea a step further. Suppose that the electron of the hydrogen atom could revolve around the nucleus in any orbit whatsoever, and not just in a limited number of orbits of which one corresponds to the normal state and the others to transient excited states. Were this true, there would be an infinity of different kinds of hydrogen atoms. This in turn would mean that hydrogen would not be the distinctive and individualistic element that it is. The same may be said about every other element. But for the laws of quantization, carbon would not be carbon as we know it, oxygen would not be oxygen, iron would not be iron and gold would not be gold. What the quantum theory explains is the distinctiveness and the individuality of the 90-odd elements of which the world is made. What a long way for a theory to have come, that started out as a theory of the recondite subject of black-body radiation!

Photon Collisions

I will mention one more attribute of light into which Planck's constant has made its way. It concerns a concept which would have been inconceivable in 1900 and was not as a matter of fact conceived until 1923: namely, that a photon, or corpuscle of light, is so much a particle that it can have an elastic collision with an electron. This phenomenon was observed and interpreted by Arthur Compton, and it is therefore known as the Compton effect.

An elastic impact between two bodies is one in which both kinetic energy and momentum are conserved. After the impact each body has a different energy and a different momentum from what it had before, but the sum of the energies of the two bodies, as well as the sum of their momenta, is still the same. What Compton did was not only to propose that such an impact could occur between a photon and an electron but to state a formula for the transfer of energy and momentum.

If a corpuscle of light has linear momentum, as is implied by the fact that light exerts pressure, the momentum of a photon of frequency ν should be $h\nu/c$, the symbol c standing for the velocity of light. Consider an elastic impact between a photon and a stationary electron. The initial energy and the initial momentum of the photon are $h\nu$ and $h\nu/c$, respectively; since the electron is stationary, its initial energy and momentum are zero. Let us suppose that the electron recoils from the impact in a direction at a certain angle with the direction in which the photon was originally traveling. Now the equations of the impact are easy to solve. One can calculate the angle of recoil of the photon, its new energy, which turns out to be less than its original energy $h\nu$ and its new momentum, which turns out to be less than its original momentum $h\nu/c$.

We can now write an equation which gives the new frequency of the photon: its value is E'/h—the new energy divided by Planck's constant. In short, what this equation and the foregoing theory say is that if the electron recoils in a certain direction, the photon goes off in a certain calculable direction and has a lesser frequency (or longer wavelength) than it had before, this new frequency also being calculable.

To test this theory, it is only necessary to set up an X-ray spectroscope to catch the photons going off in the theoretically calculated direction. When this test was actually made, by directing X-rays (photons) against a target of matter rich in electrons, Compton's theory was completely confirmed.

Now we have reached the twin climaxes of the early quantum theory: the law of composition of angular momenta and the Compton effect. We have also reached the year 1923. At this point in the story quantum theory suffers a mighty change. Its expression in terms of mathematics becomes much harder; its expression in terms of words and of analogies with concepts of the past becomes so very much harder as to verge on the impossible. But if one is interested only in the simpler applications of the theory, it is not necessary to attempt to pass through the formidable portal.