Circuit 1 Apex Path Sharing

const $P = 16$ ▷ number of partitions
const $A = 7$ ▷ height of apex tree
const $C = \lceil 1375/P \rceil$

comm_d_new $\overset{\diamond}{=}$ alloc_pub(CommDNew$_{\text{pub}}$)
partition_id $\overset{\diamond}{=}$ alloc_pub(ParitionID$_{\text{pub}}$)
partition_id_bits : $\mathbb{F}_q|_{\{0,1\}}^{|\log_2(P)|} \overset{\diamond}{=}$ le_bits_gadget(partition_id, $\log_2(P)$)

(label_partition, path_partition) $\overset{\diamond}{=}$ alloc_bintree_proof(TreeDPartitionProof$_{\text{priv}}$)
commd_d_new_calc $\overset{\diamond}{=}$ bintree_root_gadget(partition_id_bits, label_partition, path_partition)
assert_eq(commd_d_new_calc, commd_d_new) ▷ prove that label_partition leads to CommD

apex_nodes$_0$: $\mathbb{F}_q{}^{|2^A|} \overset{\diamond}{=}$ alloc(ApexTreeLeafs$_{\text{priv}}$)
for $i \in [1, A]$ **do**
  def apex_nodes$_i$: $\mathbb{F}_q{}^{|2^{A-i}|}$
  for $j \in [0, 2^{A-i})$ **do**
    apex_nodes$_i[j] \overset{\diamond}{=}$ sha_gadget((apex_nodes$_{i-1}[2j]$,apex_nodes$_{i-1}[2j+1]$))
  end for
end for
assert_eq(apex_nodes$_A[0]$, label_partition) ▷ prove that apex leafs lead to label_partition

for $c \in [0, C)$ **do**
  c_bits : $\mathbb{F}_q|_{\{0,1\}}^{|\log_2(N)-\log_2(P)|} \overset{\diamond}{=}$ challenge_gen(c) ▷ simplified challenge generation
  apex_leaf : $\mathbb{F}_q \overset{\diamond}{=}$ lookup_gadget$_{2^A}$(apex_nodes$_0$, c_bits$[\log_2(N) - \log_2(P) - A .. \log_2(N) - \log_2(P)]$)
  (label_d, path_d) $\overset{\diamond}{=}$ alloc_bintree_proof(TreeDProof$_{\text{c,priv}}$)
  apex_leaf_calc $\overset{\diamond}{=}$ bintree_root_gadget(c_bits$[0 .. \log_2(N) - \log_2(P) - A]$,label_d, path_d)
  assert_eq(apex_leaf_calc, apex_leaf) ▷ prove path from CommD leaf to apex leaf
end for

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