ALGEBRA 2 PRACTICE PACK

Polynomial Graphs, Zeros & Transformations

QUICK REFERENCE

What does intercept form tell you? In $f(x) = a(x - p)(x - q)(x - r)$, every factor hands you an x-intercept for free: $(x - 3)$ means the graph hits the x-axis at $x = 3$, and $(x + 2)$ means it hits at $x = -2$ (watch that sign flip!). The exponent on a factor is its multiplicity, and it tells you what the graph does there. A single factor crosses straight through, a squared factor like $(x - 1)^2$ touches the axis and bounces back, and a cubed factor flattens out a bit as it crosses.

Which way do the ends point? Add up the exponents on all the x's to get the degree. An even degree has the same end behavior as $y = x^2$, with both ends pointing up. An odd degree has the same end behavior as $y = x^3$, falling on the left and rising on the right. If a is negative, flip the ends upside down. Only the ends have to match, though, so the middle of the graph can wiggle a lot more than a plain parabola or cubic.

Going backward from a graph: Write one factor for each x-intercept, squaring it wherever the graph bounces. Then plug in one more point from the graph (the y-intercept is usually easiest) and solve for a. "Least degree" just means you don't add any factors beyond what the graph shows.

Where do the other zeros hide? A polynomial of degree n has exactly n zeros when you count multiplicity. The graph only shows the real ones, the x-intercepts, and a bounce counts as two. Whatever is left over is imaginary, so imaginary zeros = degree − real zeros. Imaginary zeros always come in pairs, so an odd count means it's time to recount!

How do transformations change the equation? Changes inside the parentheses move the graph left or right, the opposite way you'd expect: $f(x - 2)$ shifts right 2 and $f(x + 5)$ shifts left 5. Replacing every x with $-x$, $f(-x)$, reflects it over the y-axis. Changes outside are vertical: $f(x) + 4$ shifts up 4, $-f(x)$ reflects over the x-axis, and $a \cdot f(x)$ stretches it when $a > 1$ or shrinks it when a is between 0 and 1. Vertical changes happen after everything x does, so keep the whole function grouped as one chunk, in parentheses or factored, before you flip, stretch, or shift it.

Examples

Example 1: Graphing from Intercept Form

Graph $f(x) = (x + 2)(x - 1)(x - 3)$.

Zeros: $x = -2$, 1, and 3. Each factor shows up once, so the graph crosses at all three.
y-intercept: plug in 0 to get $(2)(-1)(-3) = 6$, so the point is (0, 6).

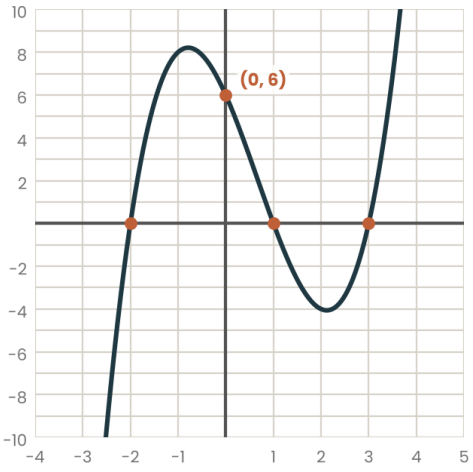
End behavior: three x's multiplied together means degree 3. That's odd, so the ends act like $y = x^3$, and since $a = 1$ is positive there's no flip: falls on the left, rises on the right. Plot the points, then connect them with a smooth curve that follows that end behavior.

A harder one: $f(x) = -(x + 1)^2(x - 2)$.

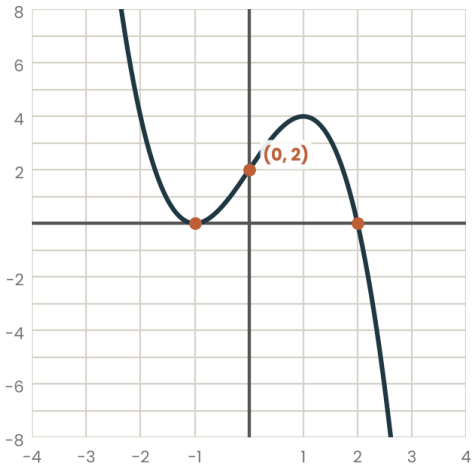
Zeros: $x = -1$ has a squared factor, so the graph bounces there. $x = 2$ has a single factor, so it crosses. **y-intercept:** $-(1)^2(-2) = 2$, so the point is (0, 2).

End behavior: the exponents add to $2 + 1 = 3$, which is odd, so the ends act like $y = x^3$. But $a = -1$ is negative, so flip them: rises on the left, falls on the right.

$f(x) = (x + 2)(x - 1)(x - 3)$

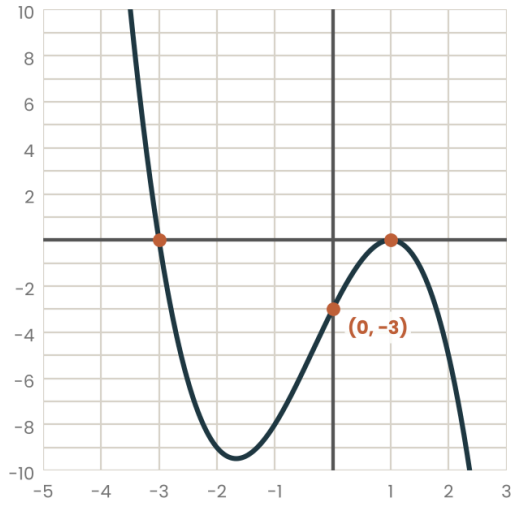


$f(x) = -(x + 1)^2(x - 2)$



Example 2: Writing a Function from Its Graph

Write a polynomial function of least degree for this graph.



Factors: the graph crosses at $x = -3$, giving $(x + 3)$. It bounces at $x = 1$, giving $(x - 1)^2$. So $f(x) = a(x + 3)(x - 1)^2$.

Find a: the graph passes through $(0, -3)$. Plug it in: $-3 = a(0 + 3)(0 - 1)^2$, which simplifies to $-3 = 3a$, so $a = -1$.

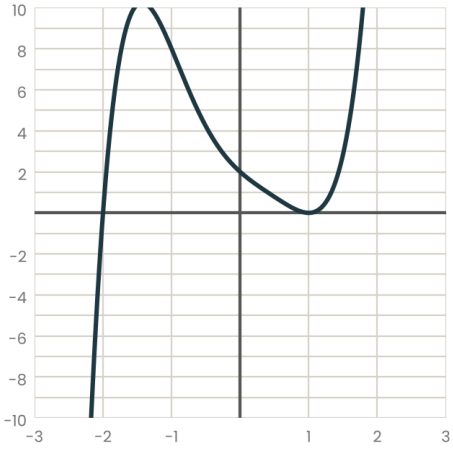
Answer: $f(x) = -(x + 3)(x - 1)^2$

Quick check: degree 3 means $y = x^3$ ends, and a negative a flips them, so the graph should rise on the left and fall on the right. It does!

Example 3: Counting Real and Imaginary Zeros



Degree 4. The graph crosses the x-axis twice, so there are 2 real zeros. That leaves $4 - 2 =$ **2 imaginary zeros.**



Degree 5. The graph crosses once (at -2) and bounces once (at 1), and the bounce counts twice, so there are $1 + 2 = 3$ real zeros. That leaves $5 - 3 =$ **2 imaginary zeros.**

Example 4: Writing a Transformed Function

$f(x) = x^3 - 2x$. Write $g(x)$, a translation of f 3 units right.

Moving right means the change goes inside, and it uses the opposite sign. Replace every x with $(x - 3)$:

$$g(x) = (x - 3)^3 - 2(x - 3)$$

A harder one: $f(x) = x^3 - 3x^2 + 4$. Write $g(x)$, a reflection of f in the x-axis followed by a translation 5 units up.

Both of these are vertical changes. They happen to the output, after the function has done all its work with x, so in the order of operations they come last. That means the whole function has to be separated from them as one chunk first.

Here's what goes wrong if you skip that. Sticking a negative in front of $x^3 - 3x^2 + 4$ gives $-x^3 - 3x^2 + 4$, and that only flips the first term!

Keep it together by factoring first. This f factors into $(x + 1)(x - 2)^2$, a single product, so the negative out front multiplies all of it: the reflection is $-(x + 1)(x - 2)^2$. Then the +5 goes on the very end, outside the flip:

$$g(x) = -(x + 1)(x - 2)^2 + 5$$

Wrapping f in parentheses works too: $g(x) = -(x^3 - 3x^2 + 4) + 5$. Notice the +4 inside the parentheses belongs to f, so the flip turns it into −4, while the +5 outside is the shift and stays positive. Both forms simplify to the same thing, $-x^3 + 3x^2 + 1$. The factored form is just nicer because it also shows you the zeros.

Warm-Up

Give each zero (cross or bounce?), the degree, and the end behavior (like $y = x^2$ or $y = x^3$, flipped or not?).

1. $f(x) = (x - 4)(x + 5)$

Zeros + cross/bounce: _____

Degree: _____ End behavior: _____

2. $f(x) = 2x(x - 3)^2$

Zeros + cross/bounce: _____

Degree: _____ End behavior: _____

3. $f(x) = -(x + 2)^3(x - 1)$

Zeros + cross/bounce: _____

Degree: _____ End behavior: _____

4. $f(x) = -3(x + 1)(x - 1)(x - 6)$

Zeros + cross/bounce: _____

Degree: _____ End behavior: _____

5. $f(x) = (x + 4)^2(x - 2)^2$

Zeros + cross/bounce:

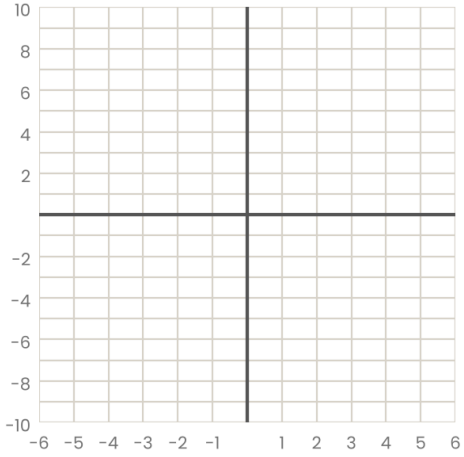
Degree: End behavior:

Core Practice

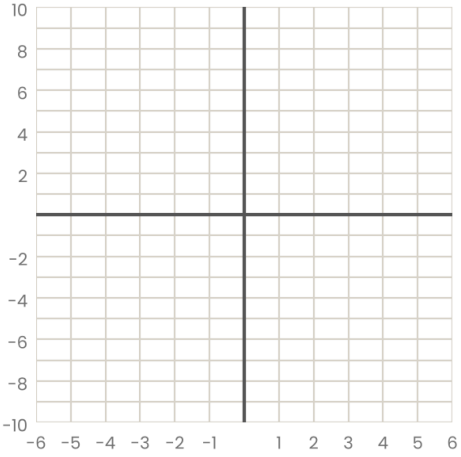
A. Graphing from Intercept Form

Plot the x-intercepts and the y-intercept, decide whether the graph crosses or bounces at each zero, use the end behavior, then sketch the graph.

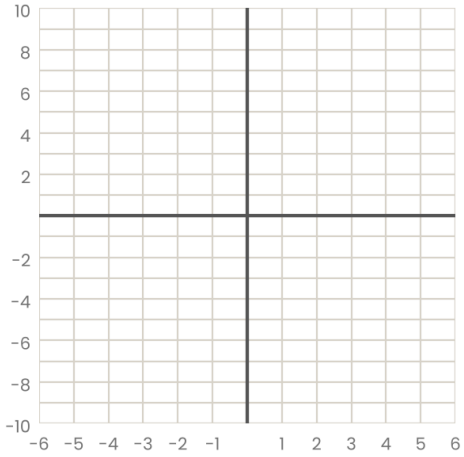
6. $f(x) = (x + 1)(x - 2)(x - 4)$



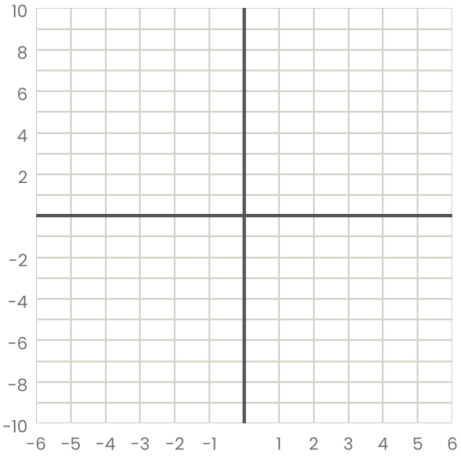
7. $f(x) = -x(x - 2)(x + 3)$



8. $f(x) = (x + 2)^2(x - 2)$



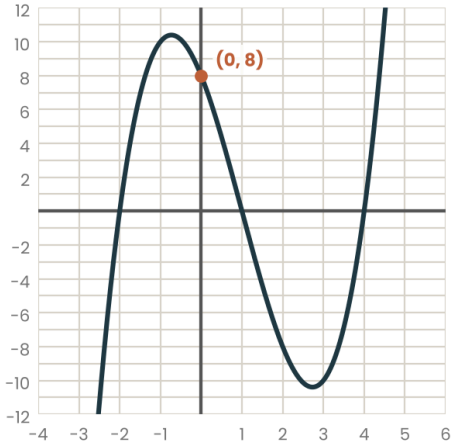
9. $f(x) = (x - 1)^2(x + 1)(x - 3)$



B. Writing a Function from Its Graph

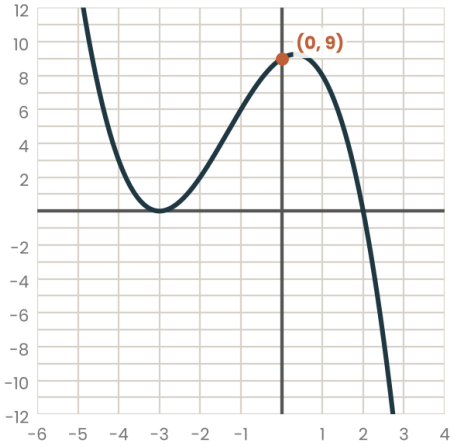
Write a polynomial function in intercept form with the least degree possible for each graph.

10.



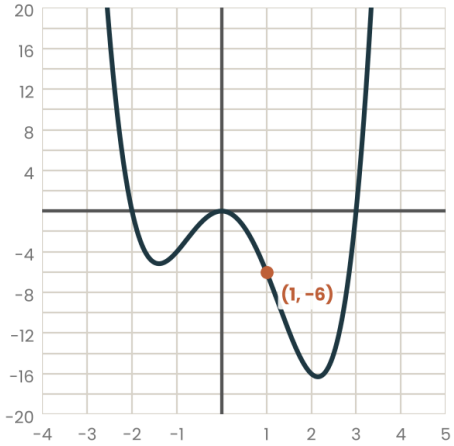
f(x) = _____

11.



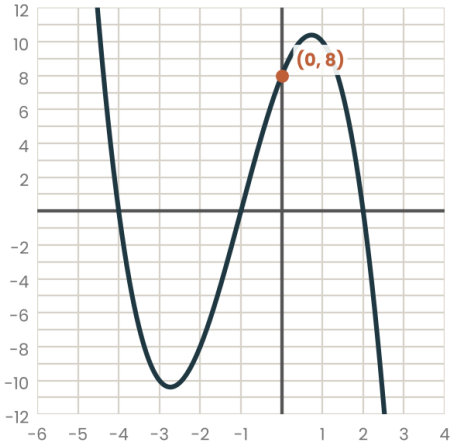
f(x) = _____

12.



f(x) = _____

13.

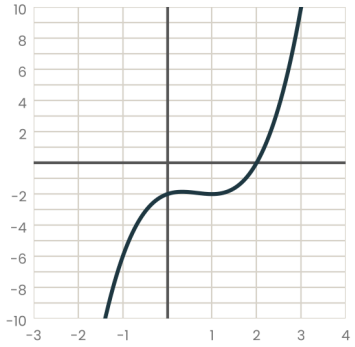


f(x) = _____

C. Real and Imaginary Zeros

The graph of a polynomial function f and its degree are given. How many real zeros and how many imaginary zeros does f have? Count a bounce as two real zeros.

14. Degree 3



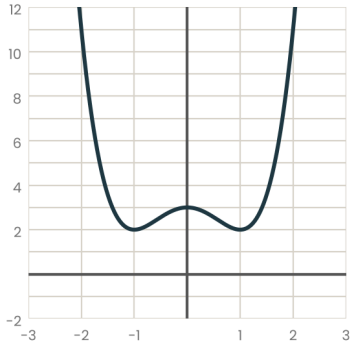
Real: Imaginary:

15. Degree 4



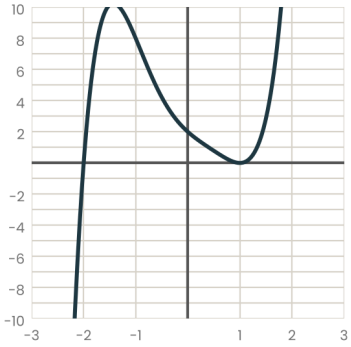
Real: Imaginary:

16. Degree 4



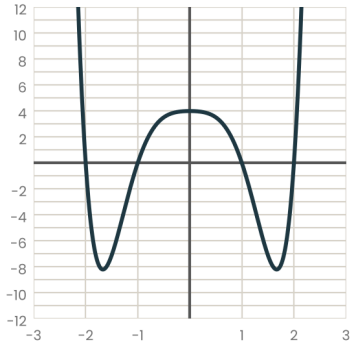
Real: Imaginary:

17. Degree 5



Real: Imaginary:

18. Degree 6



Real: Imaginary:

D. Transformations

Write a rule for g, the transformation of f described. Problem 24 works backward: describe the transformation instead.

19. $f(x) = x^3 + 2$; translation 4 units left

$g(x) =$ _____

20. $f(x) = x^4 - 3x^2$; reflection in the x-axis

$g(x) =$ _____

21. $f(x) = x^3 - 4x + 1$; reflection in the y-axis

$g(x) =$ _____

22. $f(x) = (x - 1)(x + 3)^2$; translation 2 units right and 5 units down

$g(x) =$ _____

23. $f(x) = x^4 + x$; vertical stretch by a factor of 3, followed by a translation 1 unit up

$g(x) =$ _____

24. $f(x) = x^4$ and $g(x) = -(x + 5)^4 - 2$. Describe the transformations from f to g.

_____

_____

CHALLENGE

Let $f(x) = (x + 1)(x - 2)^2$. The function g is a reflection of f in the x-axis, followed by a translation 3 units left.

a) Write g(x) in intercept form.

b) List the zeros of g and say whether the graph crosses or bounces at each one.

c) Give the degree and end behavior of g, then say how many real and imaginary zeros it has.

ANSWER KEY

Warm-Up

1. $f(x) = (x - 4)(x + 5)$ Single factors, a = 1 > 0. **Zeros 4 and −5, both cross. Degree 2 (even): ends like $y = x^2$, rising on both sides.**

2. $f(x) = 2x(x - 3)^2$ The factor x gives a zero at 0; (x − 3) is squared. **0 crosses, 3 bounces. Degree 3 (odd): ends like $y = x^3$, falling left and rising right.**

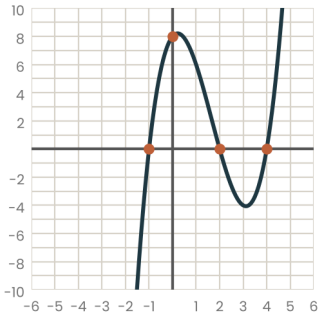
3. $f(x) = -(x + 2)^3(x - 1)$ (x + 2) is cubed; exponents add to 3 + 1 = 4; a = −1 < 0. **−2 crosses (flattening out), 1 crosses. Degree 4 (even): $y = x^2$ ends flipped, falling on both sides.**

4. $f(x) = -3(x + 1)(x - 1)(x - 6)$ Three single factors, a = −3 < 0. **−1, 1, and 6 all cross. Degree 3 (odd): $y = x^3$ ends flipped, rising left and falling right.**

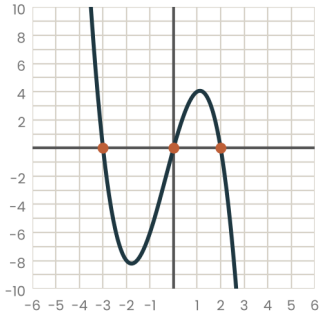
5. $f(x) = (x + 4)^2(x - 2)^2$ Both factors squared; exponents add to 4; a = 1 > 0. **−4 and 2 both bounce. Degree 4 (even): ends like $y = x^2$, rising on both sides.**

A. Graphing from Intercept Form

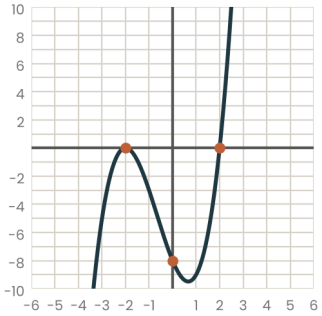
6. $f(x) = (x + 1)(x - 2)(x - 4)$ Zeros −1, 2, 4 (all cross). y-intercept (1)(−2)(−4) = 8. Degree 3, a > 0: ends like $y = x^3$, falling left and rising right.



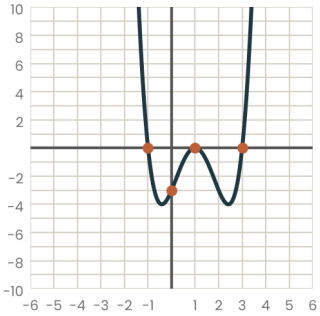
7. $f(x) = -x(x - 2)(x + 3)$ Zeros −3, 0, 2 (all cross). y-intercept 0. Degree 3, a = −1 < 0: $y = x^3$ ends flipped, rising left and falling right.



8. $f(x) = (x + 2)^2(x - 2)$ −2 bounces, 2 crosses. y-intercept $(2)^2(-2) = -8$. Degree 3, $a > 0$: ends like $y = x^3$.



9. $f(x) = (x - 1)^2(x + 1)(x - 3)$ 1 bounces, −1 and 3 cross. y-intercept $(-1)^2(1)(-3) = -3$. Degree 4, $a > 0$: ends like $y = x^2$, rising on both sides.



B. Writing a Function from Its Graph

10. *Crosses at* −2, 1, 4; *passes through* (0, 8) $f(x) = a(x + 2)(x - 1)(x - 4)$. At (0, 8): $8 = a(2)(-1)(-4) = 8a$, so $a = 1$. **$f(x) = (x + 2)(x - 1)(x - 4)$**

11. *Bounces at* −3, *crosses at* 2; *passes through* (0, 9) $f(x) = a(x + 3)^2(x - 2)$. At (0, 9): $9 = a(3)^2(-2) = -18a$, so $a = -1/2$. **$f(x) = -1/2(x + 3)^2(x - 2)$**

12. *Crosses at* −2 *and* 3, *bounces at* 0; *passes through* (1, −6) $f(x) = a{\cdot}x^2(x + 2)(x - 3)$. The y-intercept is 0, so use (1, −6): $-6 = a(1)^2(3)(-2) = -6a$, so $a = 1$. **$f(x) = x^2(x + 2)(x - 3)$**

13. *Crosses at* −4, −1, 2; *passes through* (0, 8) $f(x) = a(x + 4)(x + 1)(x - 2)$. At (0, 8): $8 = a(4)(1)(-2) = -8a$, so $a = -1$. **$f(x) = -(x + 4)(x + 1)(x - 2)$**

C. Real and Imaginary Zeros

14. *Degree* 3 Crosses once, so 1 real zero; 3 − 1 = 2. **1 real, 2 imaginary**

15. *Degree* 4 Crosses twice, so 2 real zeros; 4 − 2 = 2. **2 real, 2 imaginary**

16. *Degree* 4 Never touches the x-axis, so 0 real zeros; 4 − 0 = 4. **0 real, 4 imaginary**

17. *Degree* 5 Crosses at −2 (1) and bounces at 1 (2), so 3 real zeros; 5 − 3 = 2. **3 real, 2 imaginary**

18. *Degree* 6 Crosses at −2, −1, 1, and 2, so 4 real zeros; 6 − 4 = 2. **4 real, 2 imaginary**

D. Transformations

19. $x^3 + 2$; *4 units left* Left is an inside change with the opposite sign: replace x with (x + 4). **$g(x) = (x + 4)^3 + 2$**

20. $x^4 - 3x^2$; *reflect in the x-axis* $g(x) = -f(x) = -(x^4 - 3x^2)$. **$g(x) = -x^4 + 3x^2$**

21. $x^3 - 4x + 1$; *reflect in the y-axis* $g(x) = f(-x) = (-x)^3 - 4(-x) + 1$. **$g(x) = -x^3 + 4x + 1$**

22. $(x - 1)(x + 3)^2$; *2 right, 5 down* Replace x with (x − 2): $(x - 2 - 1)(x - 2 + 3)^2$, then subtract 5. **$g(x) = (x - 3)(x + 1)^2 - 5$**

23. $x^4 + x$; *stretch by 3, then 1 up* Stretch first: $3f(x) = 3(x^4 + x)$. Then add 1. **$g(x) = 3x^4 + 3x + 1$**

24. $f(x) = x^4$, $g(x) = -(x + 5)^4 - 2$ The negative out front flips it; (x + 5) inside moves it left; −2 outside moves it down. **Reflection in the x-axis, then a translation 5 units left and 2 units down.**

Challenge

a. *Write g(x)* Reflect: −f(x). Shift 3 left: replace x with (x + 3), so $g(x) = -(x + 3 + 1)(x + 3 - 2)^2$. **$g(x) = -(x + 4)(x + 1)^2$**

b. *Zeros of g* (x + 4) appears once; (x + 1) is squared. **−4 crosses, −1 bounces.**

c. *Degree, end behavior, zero count* Exponents add to 1 + 2 = 3, a = −1 < 0. Real zeros: −4 once and −1 twice, so 3 real; 3 − 3 = 0. **Degree 3; $y = x^3$ ends flipped (rises left, falls right); 3 real, 0 imaginary.**