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A

1. Let x be the number. $x + x^2 = 56$, $x^2 + x - 56 = 0$, $(x + 8)(x - 7) = 0$; $x + 8 = 0$, $x = -8$; $x - 7 = 0$, $x = 7$; The number is either -8 or 7.

2. Let x be the number. $x^2 - x = 72$, $x^2 - x - 72 = 0$, $(x - 9)(x + 8) = 0$; $x - 9 = 0$, $x = 9$; $x + 8 = 0$, $x = -8$; The number is either 9 or -8.

3. Let x be the number. $x^2 - 30 = x$, $x^2 - x - 30 = 0$, $(x - 6)(x + 5) = 0$; $x - 6 = 0$, $x = 6$; $x + 5 = 0$, $x = -5$ (not a pos. number); The number is 6.

4. Let x be the number. $x^2 - 42 = x$, $x^2 - x - 42 = 0$, $(x - 7)(x + 6) = 0$; $x - 7 = 0$, $x = 7$ (not a negative number); $x + 6 = 0$, $x = -6$; The number is -6.

5. Let n be the first integer. $n(n + 1) = 90$, $n^2 + n = 90$, $n^2 + n - 90 = 0$, $(n + 10)(n - 9) = 0$; $n + 10 = 0$, $n = -10$; $n - 9 = 0$, $n = 9$ (not a neg. integer); The integers are -10 and -9.

6. Let n be the first integer. $n(n + 2) = 143$, $n^2 + 2n = 143$, $n^2 + 2n - 143 = 0$, $(n + 13)(n - 11) = 0$; $n + 13 = 0$, $n = -13$ (not a pos. integer); $n - 11 = 0$, $n = 11$; The integers are 11 and 13.

7. Let n be the first integer. $n^2 + (n + 2)^2 = 340$, $n^2 + n^2 + 4n + 4 = 340$, $2n^2 + 4n + 4 = 340$, $2n^2 + 4n - 336 = 0$, $2(n^2 + 2n - 168) = 0$, $n^2 + 2n - 168 = 0$, $(n + 14)(n - 12) = 0$; $n + 14 = 0$, $n = -14$ (not a pos. integer); $n - 12 = 0$, $n = 12$; The integers are 12 and 14.

8. Let n be the first integer. $n^2 + (n + 2)^2 = 100$, $n^2 + n^2 + 4n + 4 = 100$, $2n^2 + 4n + 4 = 100$, $2n^2 + 4n - 96 = 0$, $2(n^2 + 2n - 48) = 0$, $n^2 + 2n - 48 = 0$, $(n + 8)(n - 6) = 0$; $n + 8 = 0$, $n = -8$; $n - 6 = 0$, $n = 6$ (not a neg. integer); The integers are -8 and -6.

9. Let w be the width of the rectangle. $w(w + 8) = 105$, $w^2 + 8w = 105$, $w^2 + 8w - 105 = 0$, $(w + 15)(w - 7) = 0$; $w + 15 = 0$, $w = -15$ (width cannot be neg.); $w - 7 = 0$, $w = 7$; $w + 8 = 15$; The rectangle is 7 cm by 15 cm.

10. Let w be the width of the rectangle. $w(2w - 6) = 108$, $2w^2 - 6w = 108$, $2w^2 - 6w - 108 = 0$, $2(w^2 - 3w - 54) = 0$, $w^2 - 3w - 54 = 0$, $(w - 9)(w + 6) = 0$; $w - 9 = 0$, $w = 9$; $2w - 6 = 18 - 6 = 12$; $w + 6 = 0$, $w = -6$ (width cannot be neg.); The rectangle is 9 cm by 12 cm.

11. Let w be the width of the rectangle. Let l be the length of the rectangle.

$2w + 2l = 46$, $2l = 46 - 2w$, $l = \frac{1}{2}(46 - 2w)$, $l = 23 - w$; $w(23 - w) = 126$,

$23w - w^2 = 126$, $0 = w^2 - 23w + 126$, $0 = (w - 9)(w - 14)$; $w - 9 = 0$, $w = 9$; $23 - w = 23 - 9 = 14$; $w - 14 = 0$, $w = 14$; $23 - w = 23 - 14 = 9$; The rectangle is 9 m by 14 m.

12. Let w be the width and l the length of the rectangle. $2w + 2l = 42$, $2l = 42 - 2w$,

$l = \frac{1}{2}(42 - 2w)$, $l = 21 - w$; $w(21 - w) = 104$, $21w - w^2 = 104$,

$0 = w^2 - 21w + 104$, $0 = (w - 8)(w - 13)$; $w - 8 = 0$, $w = 8$; $21 - w = 21 - 8 = 13$; $w - 13 = 0$, $w = 13$; $21 - w = 21 - 13 = 8$; The rectangle is 8 m by 13 m.

13. Let x be one of the numbers and $25 - x$ be the other number.
$x^2 + (25 - x)^2 = 313$, $x^2 + 625 - 50x + x^2 = 313$, $2x^2 - 50x + 625 = 313$,
$2x^2 - 50x + 312 = 0$, $2(x^2 - 25x + 156) = 0$, $x^2 - 25x + 156 = 0$,
$(x - 12)(x - 13) = 0$; $x - 12 = 0$, $x = 12$; $25 - x = 25 - 12 = 13$; $x - 13 = 0$,
$x = 13$; $25 - x = 25 - 13 = 12$; The numbers are 12 and 13.

14. Let x be the smaller number and $x + 5$ be the larger number. $x^2 + (x + 5)^2 = 233$,
$x^2 + x^2 + 10x + 25 = 233$, $2x^2 + 10x + 25 = 233$, $2x^2 + 10x - 208 = 0$,
$2(x^2 + 5x - 104) = 0$, $x^2 + 5x - 104 = 0$, $(x + 13)(x - 8) = 0$; $x + 13 = 0$,
$x = -13$ (not a pos. number); $x - 8 = 0$, $x = 8$; $x + 5 = 8 + 5 = 13$; The numbers are 8 and 13.

15. Let x be the amount each dimension was reduced. $(20 - x)(23 - x) =$
$20 \cdot 23 - 120$, $460 - 43x + x^2 = 460 - 120$, $460 - 43x + x^2 = 340$,
$x^2 - 43x + 120 = 0$, $(x - 40)(x - 3) = 0$; $x - 40 = 0$, $x = 40$; $20 - x =$
$20 - 40 = -20$ (width cannot be neg.); $x - 3 = 0$, $x = 3$; $20 - x = 20 - 3 = 17$;
$23 - x = 23 - 3 = 20$. The new rectangle is 17 cm by 20 cm.

16. Let w be the width and $2w$ be the length of the original rectangle.
$(2w + 4)(w - 3) = 600$, $2w^2 - 2w - 12 = 600$, $2(w^2 - w - 6) = 600$,
$w^2 - w - 6 = 300$, $w^2 - w - 306 = 0$, $(w + 17)(w - 18) = 0$; $w + 17 = 0$,
$w = -17$ (width cannot be neg.); $w - 18 = 0$, $w = 18$; $2w + 4 = 2 \cdot 18 + 4 =$
$36 + 4 = 40$; $w - 3 = 18 - 3 = 15$; The new rectangle is 40 m by 15 m.

B **17.** Let t be the time when the ball is 19.6 m high. $19.6 = (24.5)t - 4.9t^2$,
$4.9t^2 - 24.5t + 19.6 = 0$, $4.9(t^2 - 5t + 4) = 0$, $t^2 - 5t + 4 = 0$, $(t - 4)(t - 1) = 0$;
$t - 4 = 0$, $t = 4$; $t - 1 = 0$, $t = 1$; The ball reaches 19.6 m at 1 s and 4 s.

18. Let t be the time when the rocket hits the ground. $0 = 1960t - 4.9t^2$,
$0 = 4.9t(400 - t)$; $4.9t = 0$, $t = 0$ (when the rocket leaves the ground); $400 - t = 0$,
$400 = t$; The rocket hits the ground after 400 s or $6\frac{2}{3}$ min.

19. Let t be the elapsed time. $0 = 120t - 16t^2$, $0 = 8t(15 - 2t)$; $8t = 0$, $t = 0$ (when the ball was hit); $15 - 2t = 0$, $15 = 2t$, $7.5 = t$; The catcher caught the ball after 7.5 s.

20. Let t be the time until Kathy caught the apple. $40 = 56t - 16t^2$,
$16t^2 - 56t + 40 = 0$, $8(2t^2 - 7t + 5) = 0$, $2t^2 - 7t + 5 = 0$, $(t - 1)(2t - 5) = 0$;
$t - 1 = 0$, $t = 1$ (on the way up); $2t - 5 = 0$, $2t = 5$, $t = 2.5$ (on the way down);
The apple was in the air for 2.5 s.

21. Let t be the elapsed time from the firing of the flare. $1960 = 245t - 4.9t^2$,
$4.9t^2 - 245t + 1960 = 0$, $4.9(t^2 - 50t + 400) = 0$, $t^2 - 50t + 400 = 0$,
$(t - 10)(t - 40) = 0$; $t - 10 = 0$, $t = 10$ (on the way up); $t - 40 = 0$, $t = 40$
(on the way down); $40 - 10 = 30$; The flare falls past the balloon 30 s after it was fired past it.

22. Let t be elapsed time. $-98 = 39.2t - 4.9t^2$, $4.9t^2 - 39.2t - 98 = 0$,
$4.9(t^2 - 8t - 20) = 0$, $t^2 - 8t - 20 = 0$, $(t + 2)(t - 10) = 0$; $t + 2 = 0$, $t = -2$
(time cannot be neg. here); $t - 10 = 0$, $t = 10$; The ball lands after 10 s.

23. **a.** Let t be the elapsed time. $400 = 160t - 16t^2$, $16t^2 - 160t + 400 = 0$,
$16(t^2 - 10t + 25) = 0$, $t^2 - 10t + 25 = 0$, $(t - 5)^2 = 0$; $t - 5 = 0$, $t = 5$; The projectile hits 400 ft after 5 s.

b. There is only one time when the rocket is 400 ft high.

24. Let x be the width of the border. $(12 + 2x)(4 + x) = 2 \cdot 4 \cdot 12$,
$48 + 20x + 2x^2 = 96$, $2x^2 + 20x - 48 = 0$, $2(x^2 + 10x - 24) = 0$,
$x^2 + 10x - 24 = 0$, $(x + 12)(x - 2) = 0$; $x + 12 = 0$, $x = -12$ (width cannot be neg.); $x - 2 = 0$, $x = 2$; The width of the border should be 2 m.

25. Let x be the length of one of the equal sides and $20 - 2x$ be the length of the third side. $x(20 - 2x) = 48$, $20x - 2x^2 = 48$, $0 = 2x^2 - 20x + 48$, $0 = 2(x^2 - 10x + 24)$, $0 = x^2 - 10x + 24$, $0 = (x - 6)(x - 4)$; $x - 6 = 0$, $x = 6$; $20 - 2x = 20 - 2 \cdot 6 = 20 - 12 = 8$; $x - 4 = 0$, $x = 4$; $20 - 2x = 20 - 2 \cdot 4 = 20 - 8 = 12$; The pen could have dimensions of 6 m by 8 m or 4 m by 12 m.

26. Let x be the width of each path. $40x + 30x - x^2 = 325$, $70x - x^2 = 325$, $0 = x^2 - 70x + 325$, $0 = (x - 65)(x - 5)$; $x - 65 = 0$, $x = 65$ (too wide); $x - 5 = 0$, $x = 5$; The width of each path is 5 m.

27. Let x be the length of a side of the square bottom. $4(5x) + 2x^2 = 192$, $20x + 2x^2 = 192$, $2x^2 + 20x - 192 = 0$, $2(x^2 + 10x - 96) = 0$, $x^2 + 10x - 96 = 0$, $(x + 16)(x - 6) = 0$; $x + 16 = 0$, $x = -16$ (length cannot be neg.); $x - 6 = 0$, $x = 6$; $x \cdot x \cdot 5 = 6^2 \cdot 5 = 36 \cdot 5 = 180$; The volume of the box is 180 cm^3.

28. Let x be the width of the top. $2 \cdot (w)(2w) + 2 \cdot (4 \cdot w) + 2 \cdot (4 \cdot 2w) = 220$, $4w^2 + 8w + 16w = 220$, $4w^2 + 24w = 220$, $4w^2 + 24w - 220 = 0$, $4(w^2 + 6w - 55) = 0$, $w^2 + 6w - 55 = 0$, $(w + 11)(w - 5) = 0$; $w + 11 = 0$, $w = -11$ (width cannot be neg.); $w - 5 = 0$, $w = 5$; $2w = 10$; $w \cdot 2w \cdot 4 = 5 \cdot 10 \cdot 4 = 200$; The box has a volume of 200 ft^3.

29. Let x be the width of the path. $(50 - 2x)(120 - 2x) = 2 \cdot (50x) + 2 \cdot x \cdot (120 - 2x)$, $6000 - 340x + 4x^2 = 100x + 2x(120 - 2x)$, $6000 - 340x + 4x^2 = 100x + 240x - 4x^2$, $6000 - 340x + 4x^2 = 340x - 4x^2$, $8x^2 - 680x + 6000 = 0$, $8(x^2 - 85x + 750) = 0$, $x^2 - 85x + 750 = 0$, $(x - 75)(x - 10) = 0$; $x - 75 = 0$, $x = 75$; $50 - 2x = 50 - 2 \cdot 75 = 50 - 150 = -100$ (width cannot be neg.); $x - 10 = 0$, $x = 10$; $50 - 2x = 50 - 2 \cdot 10 = 50 - 20 = 30$; $120 - 2x = 120 - 2 \cdot 10 = 120 - 20 = 100$; The lawn is 30 m by 100 m.

C **30.** Let x be the length of a shorter side and let y be the length of a longer side.

$3x + 2y = 160$, $2y = 160 - 3x$, $y = \frac{1}{2}(160 - 3x)$, $y = 80 - \frac{3}{2}x$; $x\left(80 - \frac{3}{2}x\right) = 1000$, $80x - \frac{3}{2}x^2 = 1000$, $0 = \frac{3}{2}x^2 - 80x + 1000$, $0 = 2\left(\frac{3}{2}x^2 - 80x + 1000\right)$, $0 = 3x^2 - 160x + 2000$, $0 = (3x - 100)(x - 20)$; $3x - 100 = 0$, $3x = 100$, $x = \frac{100}{3}$; $y = 80 - \frac{3}{2}x = 80 - \frac{3}{2}\left(\frac{100}{3}\right) = 80 - 50 = 30$ (but $30 < \frac{100}{3}$ and x was the *shorter* side); $x - 20 = 0$, $x = 20$; $80 - \frac{3}{2}x = 80 - \frac{3}{2}(20) = 80 - 30 = 50$; The corral is 20 yd by 50 yd.

31. Let x be the length of each edge of the smaller cube. $(x + 2)^3 - x^3 = 98$, $(x + 2)(x^2 + 4x + 4) - x^3 = 98$, $x^3 + 4x^2 + 4x + 2x^2 + 8x + 8 - x^3 = 98$, $6x^2 + 12x + 8 = 98$, $6x^2 + 12x - 90 = 0$, $6(x^2 + 2x - 15) = 0$, $x^2 + 2x - 15 = 0$, $(x + 5)(x - 3) = 0$; $x + 5 = 0$, $x = -5$ (length cannot be neg.); $x - 3 = 0$, $x = 3$; $x + 2 = 5$; The smaller cube had sides of 3 cm each and the larger cube had sides of 5 cm each.

32. Let w be the width and $w + 10$ the length of the sheet. $(w - 2 \cdot 5)[(w + 10) - 2 \cdot 5)] \cdot 5 = 6000$, $(w - 10)(w + 10 - 10) \cdot 5 = 6000$, $(w - 10)5w = 6000$, $5w^2 - 50w = 6000$, $5w^2 - 50w - 6000 = 0$, $5(w^2 - 10w - 1200) = 0$, $w^2 - 10w - 1200 = 0$, $(w + 30)(w - 40) = 0$; $w + 30 = 0$, $w = -30$ (width cannot be neg.); $w - 40 = 0$, $w = 40$; $w + 10 = 40 + 10 = 50$; The original sheet was 40 cm by 50 cm.