The equation in Step 3 of Example 1 has a root that does not check because this equation does not meet the "hidden" requirements that the rug have positive length ($12 - 2x > 0$) and positive width ($9 - 2x > 0$). Usually it is easier to write only the equation and then check its roots against other conditions stated or implied in the problem.

In the next example both solutions of the equation satisfy the conditions of the problem. You can use the formula

$$h = rt - 4.9t^2$$

to obtain a good approximation of the height h (in meters) of an object t seconds after it is projected upward with an initial speed of r meters per second (m/s).

Example 2 An arrow is shot upward with an initial speed of 34.3 m/s. When will it be at a height of 49 m?

Solution

Step 1 The problem asks for the time when the arrow is 49 m high.

Step 2 Let t = the number of seconds after being shot that the arrow is 49 m high. Let h = the height of arrow = 49 m. Let r = initial speed = 34.3 m/s.

Step 3 Substitute in the formula:

$$h = rt - 4.9t^2$$
$$49 = 34.3t - 4.9t^2$$

Step 4

$$4.9t^2 - 34.3t + 49 = 0$$
$$4.9(t^2 - 7t + 10) = 0$$
$$4.9(t - 2)(t - 5) = 0$$

Completing the solution and checking the result are left for you. A calculator may be helpful.

$\therefore$ the arrow is 49 m high both 2 s and 5 s after being shot. ***Answer***

Problems

Solve.

A

1. If a number is added to its square, the result is 56. Find the number.
2. If a number is subtracted from its square, the result is 72. Find the number.
3. A positive number is 30 less than its square. Find the number.
4. A negative number is 42 less than its square. Find the number.
5. Find two consecutive negative integers whose product is 90.
6. Find two consecutive positive odd integers whose product is 143.
7. The sum of the squares of two consecutive positive even integers is 340. Find the integers.
8. The sum of the squares of two consecutive negative even integers is 100. Find the integers.

Solve.

9. The length of a rectangle is 8 cm greater than its width. Find the dimensions of the rectangle if its area is 105 cm^2.

10. The length of a rectangle is 6 cm less than twice its width. Find the dimensions of the rectangle if its area is 108 cm^2.

11. Find the dimensions of a rectangle whose perimeter is 46 m and whose area is 126 m^2. (*Hint:* Let the width be w. Use the perimeter to find the length in terms of w.)

12. Find the dimensions of a rectangle whose perimeter is 42 m and whose area is 104 m^2.

13. The sum of two numbers is 25 and the sum of their squares is 313. Find the numbers. (*Hint:* Let one of the numbers be x. Express the other number in terms of x.)

14. The difference of two positive numbers is 5 and the sum of their squares is 233. What are the numbers?

15. Originally the dimensions of a rectangle were 20 cm by 23 cm. When both dimensions were decreased by the same amount, the area of the rectangle decreased by 120 cm^2. Find the dimensions of the new rectangle.

16. Originally a rectangle was twice as long as it was wide. When 4 m were added to its length and 3 m subtracted from its width, the resulting rectangle had an area of 600 m^2. Find the dimensions of the new rectangle.

In Exercises 17–23, use the formula $h = rt - 4.9t^2$ where h is in meters and the formula $h = rt - 16t^2$ where h is in feet. A calculator may be helpful.

B

17. A ball is thrown upward with an initial speed of 24.5 m/s. When is it 19.6 m high? (Two answers)

18. A rocket is fired upward with an initial speed of 1960 m/s. After how many minutes does it hit the ground?

19. A batter hit a baseball upward with an initial speed of 120 ft/s. How much later did the catcher catch it?

20. Mitch tossed an apple to Kathy, who was on a balcony 40 ft above him, with an initial speed of 56 ft/s. Kathy missed the apple on its way up, but caught it on its way down. How long was the apple in the air?

21. A signal flare is fired upward with initial speed 245 m/s. A stationary balloonist at a height of 1960 m sees the flare pass on the way up. How long after this will the flare pass the balloonist again on the way down?

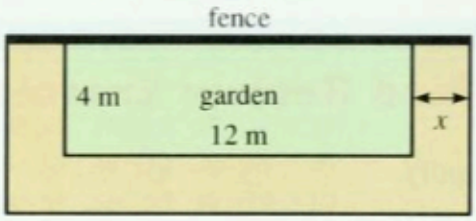


22. A ball is thrown upward from the top of a 98 m tower with initial speed 39.2 m/s. How much later will it hit the ground? (*Hint:* Consider the top of the tower as level zero. If h is the height of the ball above the top of the tower, then $h = -98$ when the ball hits the ground.)

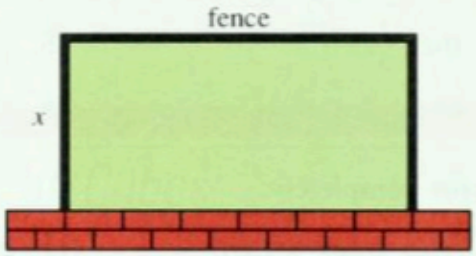
23. A rocket is fired upward with an initial velocity of 160 ft/s.
a. When is the rocket 400 ft high?
b. How do you know that 400 ft is the greatest height the rocket reaches?

Solve.

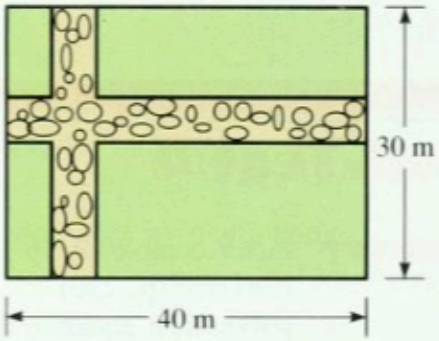
24. A garden plot 4 m by 12 m has one side along a fence as shown at the right. The area of the garden is to be doubled by digging a border of uniform width on the other three sides. What should the width of the border be?



25. Vanessa built a rectangular pen for her dogs. She used an outside wall of the garage for one of the sides of the pen. She had to buy 20 m of fencing in order to build the other sides of the pen. Find the dimensions of the pen if its area is 48 m^2.



26. A rectangular garden 30 m by 40 m has two paths of equal width crossing through it as shown. Find the width of each path if the total area covered by the paths is 325 m^2.



27. A box has a square bottom and top and is 5 cm high. Find its volume if its total surface area is 192 cm^2.

28. The bottom and top of a box are rectangles twice as long as they are wide. Find the volume of the box if it is 4 ft high and has a total surface area of 220 ft^2.

29. A 50 m by 120 m park consists of a rectangular lawn surrounded by a path of uniform width. Find the dimensions of the lawn if its area is the same as the area of the path. (*Hint:* Let x = the width of path.)

C **30.** The Parkhursts used 160 yd of fencing to enclose a rectangular corral and to divide it into two parts by a fence parallel to one of the shorter sides. Find the dimensions of the corral if its area is 1000 yd^2.

31. Each edge of one cube is 2 cm longer than each edge of another cube. The volumes of the cubes differ by 98 cm^3. Find the lengths of the edges of each cube.