ALGEBRA 2 PRACTICE PACK

Polynomials: Zeros, Division & Graphs

QUICK REFERENCE

Irrational zeros travel in pairs. If $2 + \sqrt{3}$ is a zero of a polynomial with whole-number coefficients, then its conjugate $2 - \sqrt{3}$ has to be a zero too. So when a problem hands you two zeros, it may secretly be giving you three!

The substitution trick for conjugate pairs. The two factors look like $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$, which seems messy at first. Let $u = x - 2$, and it turns into $(u - \sqrt{3})(u + \sqrt{3}) = u^2 - 3$, a plain difference of squares. Then swap $x - 2$ back in for u: $(x - 2)^2 - 3 = x^2 - 4x + 1$. Do a few and you'll spot the pattern: zeros of $a \pm \sqrt{b}$ always give $x^2 - 2ax + (a^2 - b)$.

Where does the leading coefficient go? Build the polynomial with a leading coefficient of 1 first, then multiply every single term by a at the very end. One step, saved for last.

Hunting zeros with synthetic division. List the possible rational zeros: ± (factors of the constant term) ÷ (factors of the leading coefficient). Test them until one leaves a remainder of 0. The bottom row is your new, smaller polynomial, so keep testing that one until you're down to a quadratic, then factor it or use the quadratic formula. Missing a power? Write a 0 in its spot.

Long division needs placeholders. Before dividing, rewrite the dividend with every power present: $x^4 + 2x - 3$ becomes $x^4 + 0x^3 + 0x^2 + 2x - 3$. Do the same for the divisor. Those zeros keep your columns lined up so like terms always subtract from like terms.

Reading a polynomial graph. Each x-intercept r gives you a factor $(x - r)$. If the graph crosses straight through, that factor's power is odd (usually 1). If it touches the axis and turns back (a bounce), the power is 2. Write $f(x) = a$(factors), plug in the labeled point, and solve for a.

Examples

Example 1: Irrational zeros with a leading coefficient

Write a polynomial of least degree with leading coefficient -2 and zeros 1 and $3 + \sqrt{2}$.

The conjugate $3 - \sqrt{2}$ is also a zero, so the degree is 3.

Conjugate pair: $(x - 3 - \sqrt{2})(x - 3 + \sqrt{2})$. Let $u = x - 3$: $(u - \sqrt{2})(u + \sqrt{2}) = u^2 - 2$.

Swap back: $(x - 3)^2 - 2 = x^2 - 6x + 9 - 2 = x^2 - 6x + 7$.

Multiply by the other factor: $(x - 1)(x^2 - 6x + 7) = x^3 - 7x^2 + 13x - 7$.

Apply $a = -2$ last: **$f(x) = -2x^3 + 14x^2 - 26x + 14$**

Example 2: Several rounds of synthetic division

Find all zeros of $f(x) = x^4 - 2x^3 - 6x^2 + 8x + 8$.

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8$.

Test 2 on 1, −2, −6, 8, 8. Bottom row: 1, 0, −6, −4 with remainder 0. New polynomial: $x^3 + 0x^2 - 6x - 4$.

Test −2 on 1, 0, −6, −4 (keep that 0!). Bottom row: 1, −2, −2 with remainder 0. New polynomial: $x^2 - 2x - 2$.

This quadratic doesn't factor, so use the quadratic formula: $x = 1 \pm \sqrt{3}$.

Zeros: $2, -2, 1 + \sqrt{3}, 1 - \sqrt{3}$

Example 3: Long division with missing powers

Divide $(x^4 + 2x - 3) \div (x^2 - 1)$.

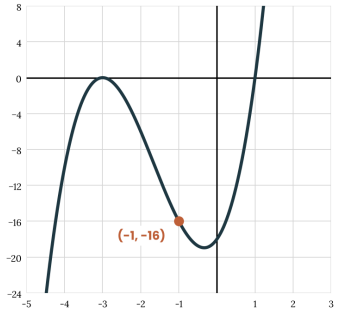
Fill in placeholders: $(x^4 + 0x^3 + 0x^2 + 2x - 3) \div (x^2 + 0x - 1)$.

$x^4 \div x^2 = x^2$. Multiply: $x^4 + 0x^3 - x^2$. Subtract: $x^2 + 2x - 3$.

$x^2 \div x^2 = 1$. Multiply: $x^2 + 0x - 1$. Subtract: $2x - 2$.

Quotient: $x^2 + 1$, Remainder: $2x - 2$

Example 4: Writing a polynomial from its graph



The graph bounces at $x = -3$ and crosses at $x = 1$.

Bounce means a squared factor: $f(x) = a(x + 3)^2(x - 1)$.

Plug in the labeled point $(-1, -16)$:

$-16 = a(2)^2(-2) = -8a$, so $a = 2$.

Expand: $2(x^2 + 6x + 9)(x - 1)$

$f(x) = 2x^3 + 10x^2 + 6x - 18$

Warm-Up: The Substitution Trick

Expand each product by letting u equal the part in front of the radical. Write your final answer in standard form, and watch for a pattern as you go!

1. $(x - 3 - \sqrt{2})(x - 3 + \sqrt{2})$

= _____

2. $(x + 1 - \sqrt{5})(x + 1 + \sqrt{5})$

= _____

3. $(x - 4 - \sqrt{7})(x - 4 + \sqrt{7})$

= _____

4. $(x + 2 - \sqrt{6})(x + 2 + \sqrt{6})$

= _____

5. Pattern check: without expanding, use what you noticed in 1 to 4 to write $(x - 5 - \sqrt{3})(x - 5 + \sqrt{3})$ in standard form. Then explain the pattern in one sentence.

= _____

Pattern: _____

_____

Core Practice

A. Writing Polynomials from Irrational Zeros

Write a polynomial of least degree in standard form with leading coefficient a and the given zeros. Don't forget the conjugates!

6. a = 1; zeros: $4, 1 + \sqrt{2}$

f(x) = _____

7. a = 2; zeros: $-2, 3 - \sqrt{5}$

f(x) = _____

8. a = −3; zeros: $-1, \sqrt{7}$

f(x) = _____

9. a = −2; zeros: $5, -4 + \sqrt{3}$

f(x) = _____

10. $a = -1$; zeros: $2, -2, 1 - \sqrt{6}$

11. $a = 2$; zeros: $1 + \sqrt{2}, 3 - \sqrt{3}$

f(x) = _____

f(x) = _____

B. Finding All Zeros with Synthetic Division

List the possible rational zeros, then use synthetic division as many times as you need until you reach a quadratic. Finish by factoring or using the quadratic formula.

12. $f(x) = x^3 - 7x - 6$

Possible rational zeros: _____

All zeros: _____

13. $f(x) = x^4 - 5x^3 + x^2 + 11x + 4$

Possible rational zeros: _____

All zeros: _____

14. $f(x) = 2x^4 - 3x^3 - 21x^2 + 17x - 3$

Possible rational zeros: _____

All zeros: _____

15. $f(x) = x^4 - x^3 + 7x^2 - 9x - 18$

Possible rational zeros: _____

All zeros: _____

16. $f(x) = x^5 - 6x^4 + 4x^3 + 20x^2 - 33x + 14$

Possible rational zeros: _____

All zeros: _____

C. Polynomial Long Division

Use long division. Rewrite the dividend (and divisor) with placeholder terms first, then give the quotient and remainder.

17. $(2x^4 + 3x - 5) \div (x^2 + 1)$

Rewritten: _____

Quotient: _____ **Remainder:** _____

18. $(x^5 - 3x^2 + 4) \div (x^2 - x + 2)$

Rewritten: _____

Quotient: _____ **Remainder:** _____

19. $(6x^4 - 5x^2 + x - 8) \div (2x^2 + 1)$

Rewritten: _____

Quotient: _____ **Remainder:** _____

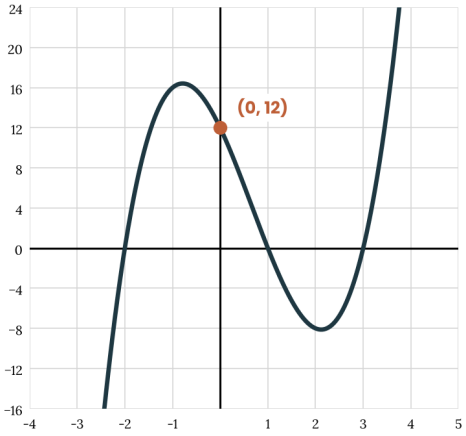
20. $(3x^5 - 2x^3 + 7) \div (x^2 + 2)$

Rewritten: _____

Quotient: _____ **Remainder:** _____

D. Writing a Polynomial from Its Graph

Write the polynomial of least degree shown in each graph. Use the labeled point to find a, then write f(x) in standard form.



21. Zeros (note bounce or cross):

_____

f(x) = a(_____

Solve for a:

f(x) = _____



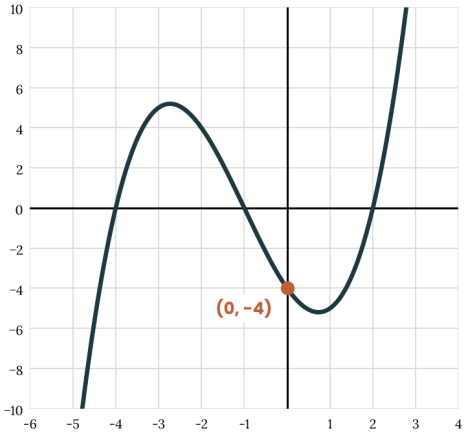
22. Zeros (note bounce or cross):

_____

f(x) = a(_____

Solve for a:

f(x) = _____



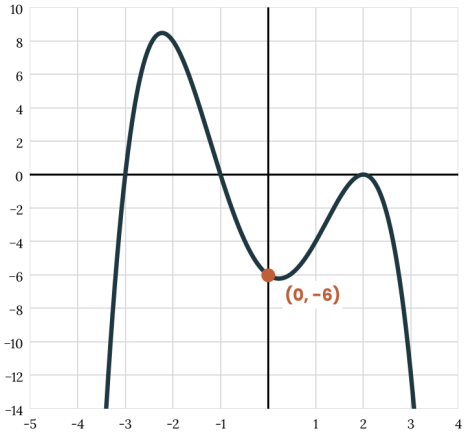
23. Zeros (note bounce or cross):

_____

f(x) = a(_____

Solve for a:

f(x) = _____



24. Zeros (note bounce or cross):

_____

f(x) = a(_____

Solve for a:

f(x) = _____

CHALLENGE

25. A cubic polynomial has zeros 3 and $1 - \sqrt{5}$, and its graph passes through $(1, -20)$. Find f(x) in standard form.

f(x) = _____

26. You're told that $2 + \sqrt{3}$ is a zero of $f(x) = x^4 - 3x^3 - 9x^2 + 25x - 6$. Find all the other zeros without any synthetic division. (Hint: build the conjugate pair's quadratic, then use long division.)

All zeros: _____

ANSWER KEY

Warm-Up

1. $(x - 3 - \sqrt{2})(x - 3 + \sqrt{2})$ | $u = x - 3$: $u^2 - 2 = (x - 3)^2 - 2$ **$x^2 - 6x + 7$**

2. $(x + 1 - \sqrt{5})(x + 1 + \sqrt{5})$ | $u = x + 1$: $u^2 - 5 = (x + 1)^2 - 5$ **$x^2 + 2x - 4$**

3. $(x - 4 - \sqrt{7})(x - 4 + \sqrt{7})$ | $u = x - 4$: $u^2 - 7 = (x - 4)^2 - 7$ **$x^2 - 8x + 9$**

4. $(x + 2 - \sqrt{6})(x + 2 + \sqrt{6})$ | $u = x + 2$: $u^2 - 6 = (x + 2)^2 - 6$ **$x^2 + 4x - 2$**

5. $(x - 5 - \sqrt{3})(x - 5 + \sqrt{3})$ | Pattern: for zeros $a \pm \sqrt{b}$, the middle coefficient is $-2a$ (negative of the sum of the zeros) and the constant is $a^2 - b$ (the product of the zeros). Here $-2(5) = -10$ and $25 - 3 = 22$. **$x^2 - 10x + 22$**

A. Writing Polynomials from Irrational Zeros

6. $a = 1$; *zeros* $4, 1 + \sqrt{2}$ | Pair: $(x - 1)^2 - 2 = x^2 - 2x - 1$. Then $(x - 4)(x^2 - 2x - 1)$. **$f(x) = x^3 - 6x^2 + 7x + 4$**

7. $a = 2$; *zeros* $-2, 3 - \sqrt{5}$ | Pair: $(x - 3)^2 - 5 = x^2 - 6x + 4$. $(x + 2)(x^2 - 6x + 4) = x^3 - 4x^2 - 8x + 8$, then $\times 2$. **$f(x) = 2x^3 - 8x^2 - 16x + 16$**

8. $a = -3$; *zeros* $-1, \sqrt{7}$ | Here $a = 0$ in the pattern, so the pair is just $x^2 - 7$. $(x + 1)(x^2 - 7) = x^3 + x^2 - 7x - 7$, then $\times (-3)$. **$f(x) = -3x^3 - 3x^2 + 21x + 21$**

9. $a = -2$; *zeros* $5, -4 + \sqrt{3}$ | Pair: $(x + 4)^2 - 3 = x^2 + 8x + 13$. $(x - 5)(x^2 + 8x + 13) = x^3 + 3x^2 - 27x - 65$, then $\times (-2)$. **$f(x) = -2x^3 - 6x^2 + 54x + 130$**

10. $a = -1$; *zeros* $2, -2, 1 - \sqrt{6}$ | $(x - 2)(x + 2) = x^2 - 4$. Pair: $(x - 1)^2 - 6 = x^2 - 2x - 5$. $(x^2 - 4)(x^2 - 2x - 5) = x^4 - 2x^3 - 9x^2 + 8x + 20$, then $\times (-1)$. **$f(x) = -x^4 + 2x^3 + 9x^2 - 8x - 20$**

11. $a = 2$; *zeros* $1 + \sqrt{2}, 3 - \sqrt{3}$ | Two pairs: $x^2 - 2x - 1$ and $(x - 3)^2 - 3 = x^2 - 6x + 6$. Product: $x^4 - 8x^3 + 17x^2 - 6x - 6$, then $\times 2$. **$f(x) = 2x^4 - 16x^3 + 34x^2 - 12x - 12$**

B. Finding All Zeros with Synthetic Division

12. $x^3 - 7x - 6$ | Possible: $\pm 1, \pm 2, \pm 3, \pm 6$. Use 1, 0, −7, −6 (placeholder for x^2). Test −1: 1, −1, −6, R 0. Then $x^2 - x - 6 = (x - 3)(x + 2)$. **Zeros: −1, −2, 3**

13. $x^4 - 5x^3 + x^2 + 11x + 4$ | Possible: $\pm 1, \pm 2, \pm 4$. Test −1: 1, −6, 7, 4, R 0. Test 4: 1, −2, −1, R 0. Then $x^2 - 2x - 1 = 0$ gives $x = 1 \pm \sqrt{2}$. (Look familiar? Compare with #6!) **Zeros: $-1, 4, 1 + \sqrt{2}, 1 - \sqrt{2}$**

14. $2x^4 - 3x^3 - 21x^2 + 17x - 3$ | Possible: $\pm 1, \pm 3, \pm 1/2, \pm 3/2$. Test −3: 2, −9, 6, −1, R 0. Test 1/2: 2, −8, 2, R 0. Then $2x^2 - 8x + 2 = 2(x^2 - 4x + 1) = 0$ gives $x = 2 \pm \sqrt{3}$. **Zeros: $-3, 1/2, 2 + \sqrt{3}, 2 - \sqrt{3}$**

15. $x^4 - x^3 + 7x^2 - 9x - 18$ | Possible: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$. Test −1: 1, −2, 9, −18, R 0. Test 2: 1, 0, 9, R 0. Then $x^2 + 9 = 0$ gives $x^2 = -9$. **Zeros: $-1, 2, 3i, -3i$**

16. $x^5 - 6x^4 + 4x^3 + 20x^2 - 33x + 14$ | Possible: $\pm 1, \pm 2, \pm 7, \pm 14$. Test 1: 1, −5, −1, 19, −14, R 0. Test 1 again: 1, −4, −5, 14, R 0. Test −2: 1, −6, 7, R 0. Then $x^2 - 6x + 7 = 0$ gives $x = 3 \pm \sqrt{2}$. **Zeros: 1 (twice), $-2, 3 + \sqrt{2}, 3 - \sqrt{2}$**

C. Polynomial Long Division

17. $(2x^4 + 0x^3 + 0x^2 + 3x - 5) \div (x^2 + 0x + 1)$ | $2x^2$: subtract $2x^4 + 2x^2$, leaving $-2x^2 + 3x - 5$. Then -2: subtract $-2x^2 - 2$, leaving $3x - 3$. **Q: $2x^2 - 2$, R: $3x - 3$**

18. $(x^5 + 0x^4 + 0x^3 - 3x^2 + 0x + 4) \div (x^2 - x + 2)$ | x^3 leaves $x^4 - 2x^3 - 3x^2$. Then x^2 leaves $-x^3 - 5x^2 + 0x$. Then $-x$ leaves $-6x^2 + 2x + 4$. Then -6 leaves $-4x + 16$. **Q: $x^3 + x^2 - x - 6$, R: $-4x + 16$**

19. $(6x^4 + 0x^3 - 5x^2 + x - 8) \div (2x^2 + 0x + 1)$ | $3x^2$: subtract $6x^4 + 3x^2$, leaving $-8x^2 + x - 8$. Then -4: subtract $-8x^2 - 4$, leaving $x - 4$. **Q: $3x^2 - 4$, R: $x - 4$**

20. $(3x^5 + 0x^4 - 2x^3 + 0x^2 + 0x + 7) \div (x^2 + 0x + 2)$ | $3x^3$: subtract $3x^5 + 6x^3$, leaving $-8x^3 + 0x^2 + 0x + 7$. Then $-8x$: subtract $-8x^3 - 16x$, leaving $16x + 7$. **Q: $3x^3 - 8x$, R: $16x + 7$**

D. Writing a Polynomial from Its Graph

21. *Crosses at* $-2, 1, 3$; *point* $(0, 12)$ | $f(x) = a(x + 2)(x - 1)(x - 3)$. $12 = a(2)(-1)(-3) = 6a$, so $a = 2$. **$f(x) = 2x^3 - 4x^2 - 10x + 12$**

22. *Bounces at* -1, *crosses at* 2; *point* $(1, 8)$ | $f(x) = a(x + 1)^2(x - 2)$. $8 = a(4)(-1) = -4a$, so $a = -2$. **$f(x) = -2x^3 + 6x + 4$**

23. *Crosses at* $-4, -1, 2$; *point* $(0, -4)$ | $f(x) = a(x + 4)(x + 1)(x - 2)$. $-4 = a(4)(1)(-2) = -8a$, so $a = 1/2$. **$f(x) = 1/2x^3 + 3/2x^2 - 3x - 4$**

24. *Crosses at* $-3, -1$, *bounces at* 2; *point* $(0, -6)$ | $f(x) = a(x + 3)(x + 1)(x - 2)^2$. $-6 = a(3)(1)(4) = 12a$, so $a = -1/2$. **$f(x) = -1/2x^4 + 9/2x^2 - 2x - 6$**

Challenge

25. *Cubic*; *zeros* $3, 1 - \sqrt{5}$; *through* $(1, -20)$ | $1 + \sqrt{5}$ is also a zero. Pair: $(x - 1)^2 - 5 = x^2 - 2x - 4$. $f(x) = a(x - 3)(x^2 - 2x - 4)$. Plug in $x = 1$: $-20 = a(-2)(-5) = 10a$, so $a = -2$. **$f(x) = -2x^3 + 10x^2 - 4x - 24$**

26. $2 + \sqrt{3}$ *is a zero of* $x^4 - 3x^3 - 9x^2 + 25x - 6$ | $2 - \sqrt{3}$ is also a zero, so $x^2 - 4x + 1$ is a factor. Long division: quotient $x^2 + x - 6$, remainder 0. Then $x^2 + x - 6 = (x + 3)(x - 2)$. **Zeros: $2 + \sqrt{3}$, $2 - \sqrt{3}$, -3, 2**